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Effects of Cylindrical Pillars in a Reverberation Chamber

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IN the reverberation theory, as ordinarily applied, the behavior of sound within a closed space is treated from a purely statistical point of view. Energy is assumed to be poured into the space from the source and is absorbed through dissipative processes mainly at the boundaries. The wave characteristics of sound are ignored, and the problem is treated using the methods of the kinetic theory of gases. The assumption of a "diffuse distribution" of sound energy is made. This calls for a uniform intensity throughout the space and a random direction of energy flow, i.e., all directions of flow are equally probable throughout the region. It is further assumed that the energy incident upon any portion of the bounding surface is proportional to the superficial area of that surface.

While it is true that in large rooms with distributed absorption these assumptions do not depart too widely from the factual situation, yet it has come to be recognized that in rooms whose dimensions are only a small number of wave-lengths of the sound, there exists a wide divergence between the facts and the theoretical assumptions. Unfortunately this situation exists in considerable degree in reverberation chambers now in use for the measurement of the absorption coefficients of materials, with the regrettable result that the numerical values which we obtain and which we should like to consider as measures of the intrinsic absorptivities of materials turn out to be dependent upon a number of other factors which vary with the size of reverberation chambers, its relative dimensions and the area of the test sample.[1]

While it is somewhat foreign to the main purpose of this paper, it may be worth while to consider the fundamental physics involved in the problem. The air inclosed in the reverberation chamber is essentially a three-dimensional elastic body and as such has its own natural modes of vibration. These modes are determined by the mechanical constants of the mass of air and the boundary condition that at a perfectly reflecting surface the normal derivative of the velocity potential is zero. In the case of a rectangular room, the frequencies of these natural modes are given by the well-known Rayleigh formula. It is well to remember that we do not escape the existence of these normal modes by departing from the rectangular shape. H. A.

[1] P. E. Sabine, J. Acous. Soc. Am. **6**, 239 (1935). Morris and Nixon, J. Acous. Soc. Am. **8**, 81 (1936). Meyer, J. Acous. Soc. Am. **8**, 155 (1937).

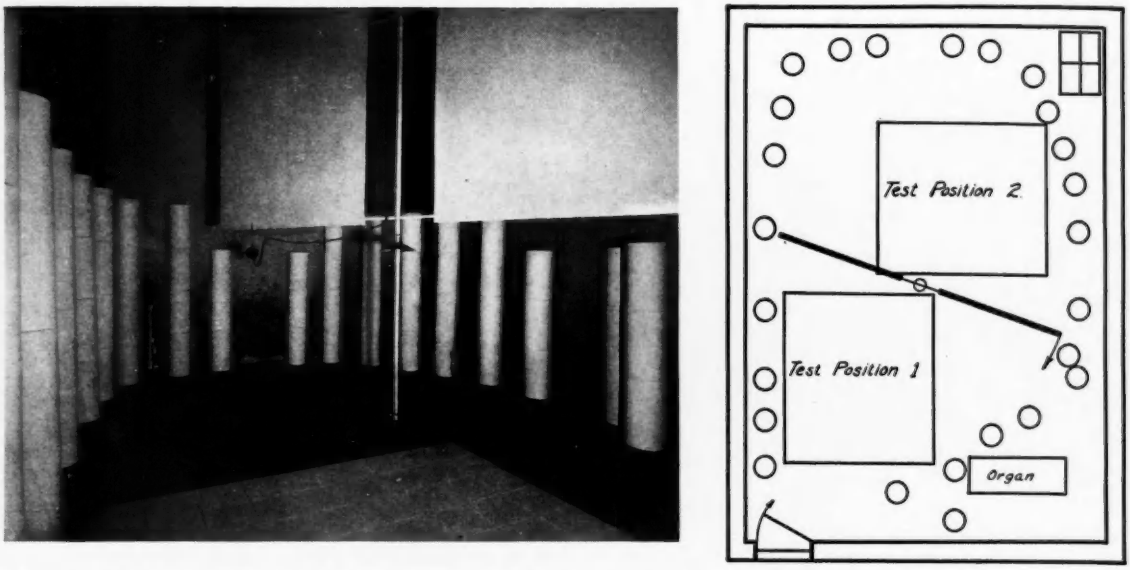

FIG. 1. Distribution of pillars 12 inches in diameter and 2 to 4 feet in height in reverberation chamber 27×19.7×19.1 feet.

Lorentz has shown that the very high frequency characteristics in which a three-dimensional body may vibrate is independent of the shape and will depend only upon the volume and the elastic properties. Building our sound chambers with no two walls parallel thus does not free us from the disturbing influence of the natural frequencies of the room. As Meyer has pointed out, we do not get rid of "Eigen periods" in nonrectangular rooms. We only lose easy means of calculating them.

The behavior of sound within a room then is simply an extension to the three-dimensional case of the less complicated phenomenon of the forced vibration of a mechanical system involving mass, elasticity and damping, and with one degree of freedom in the direction of the applied force. In the one-dimensional case, the displacement x is given by the familiar equation

$$x=A_0e^{-kT}(\sin\ \omega_0t+\varphi_1)+(F/\omega z)\ \sin\ (\omega t-\varphi_2),$$

in which the first term gives the displacement due to the transient natural frequency vibration and the second term, that due to the driving frequency.

In the initial stage the motion is the resultant of the free and the forced vibration. After a short interval the transient is damped out and the system vibrates at the forcing frequency. During the build-up period we have a complicated motion resembling the phenomenon of beats between the natural and the driving frequency. When driving ceases, due to the energy stored up in the system oscillation persists, and again there is a transition period during which the motion is complicated by the shift from the driving to the natural frequency. If the damping is small the motion in the late stages will be at the natural frequency of the system.

Now the same sort of thing may be assumed to occur when a source of sound is started and stopped in a room. There is the important difference however, that in the three-dimensional case, ω_0 ($2\pi\times$ the natural frequency) is a multiple valued quantity and that if the dimensions of the room measured in wave-lengths are large, the natural frequencies will be closely spaced in the frequency spectrum so that under these conditions any driving frequency will be close to one or more natural frequencies. Three results follow: first, the response of the room will be nearly independent of frequency; and second, the decay will be very nearly logarithmic in a room whose surfaces are highly reflective; and third, the rate of decay will be uniform throughout the room provided the absorptivity of the walls is uniform. These desirable conditions, however, are not present in rooms no larger than our present reverberation chambers for frequencies of 500 cycles or less. Here the natural

frequencies are more widely spaced in the frequency spectrum, the response of the room varies markedly with frequency, and the intensity fluctuates widely from the logarithmic law during the decay period, as a result of the interference between the driving and natural frequencies. From the standpoint of mathematical treatment the situation resembles closely that which exists in the field of subatomic physics in which the electron presents both particle and wave aspects. In rooms large in comparison with the wave-length of sound the wave characteristics of sound may be disregarded and the kinetic picture fits the facts. In *small rooms* which unfortunately include our reverberation chambers, the simpler theory is complicated by wave phenomena. These complications result (a) in marked fluctuations in intensity during the decay with resultant variations in the experimental determinations of the rate; (b) in a change in decay rate with change of position of a highly absorbent test sample; and (c) a possible influence on the so-called "area effect" i.e., the relatively greater change on the decay rate produced by small samples than by large samples.

The present investigation was undertaken with a view to determining what effects, if any, would result from placing a considerable number of nonabsorbent cylindrical pillars in a reverberation chamber. These pillars were 12 inches in diameter and varied in height from 2 to 8 feet. They were made of hard plaster applied on metal lath. The random placement in the reverberation chamber is shown in Fig. 1. The idea back of the investigation is that, due to scattering from the cylindrical surfaces, the persistence of normal modes of vibration would be decreased. If so, then the ideal of a diffuse distribution would be more nearly approximated and the complications due to stationary wave correspondingly lessened. Meyer[1] has tried the effect of acoustically roughening the walls of a rectangular room by corrugating the surface. This was in a small room, volume 13 cubic meter and the depths of the corrugation were comparable with the wave-length of the 7000-cycle tone employed. He reports that the undesired effects of reflection from smooth surfaces were diminished but not eliminated. In the present experiment the conditions were somewhat different in the fact that the cylinders were placed away from the walls, thus allowing scattering over a wider angle than if they served simply as corrugations of the reflecting wall surface. The percentages of the scattered energy of a plane wave incident upon a cylindrical surface 12 inches in diameter, computed for different frequencies are as follows:[2]

Frequency	128	256	512	1024	2048
Percent scattered	0.18	0.40	0.63	0.80	0.90

A larger percentage of scattering at the lower frequency is of course desirable, but in view of the tentative nature of the experiment and the difficulties and expense involved in building larger pillars the smaller ones were used.

The effects investigated were the following:

(a) On the deviations of single measurements of the decay from the mean of a large number of measurements.
(b) On the variation in measured coefficients due to variation in the area of test sample.
(c) On variations in the rate of decay caused by changing the position of an absorbent area.
(d) On the measured value of the coefficients of absorption of materials.

The procedure was to make careful determination of all the quantities involved before and after the pillars were installed. The investigation extended over the period from April 21 to August 5. Frequent checks of the electrical factors of the measurements were made to insure as nearly constant conditions as possible. The study involved some 21,000 separate observations. All measurements were made with a stationary sound source giving a ± 20 cycle warble tone and with the pick up microphone mounted on the moving vanes.

Effect on Precision of Measurements of Decay Rates

Individual measurements of the time required for the intensity level in the reverberation chamber to decrease from a fixed initial level to the cut-off level of the pick-up circuit show deviations from the mean of a large number of observations. These deviations arise first from a lack of absolute constancy in the amplification of the pick-up circuit, and second, from the fluctuation of the intensity above and below the

[2] Morse, *Vibration and Sound* (McGraw-Hill), p. 265.

theoretic logarithmic decay. It is of course not possible to separate these two sources of deviation, but electrical checks on the measuring equipment indicate that at the lower frequencies the latter is far the greater of the two. Experiment showed that, at a fixed frequency, the absolute values of the deviations are independent of the initial intensity but that there is a marked correlation with the frequency of the sound. The average values shown in Table I are taken from measurements made for each of five different initial levels at each frequency.

Column 1 gives the rate of decay in db/sec.
Column 2 is the average deviation in seconds of a single timing from the average of 10 timings.
Column 3 is the average deviation of the average of one set of 10 timings from the grand average of five such sets.
Column 4 is the average deviation shown in 3, multiplied by the decay rate. It is a measure of the fluctuation in the intensity level in the neighborhood of the threshold level of the pick-up system. The smaller this value the smaller the ripples in the decay curve, if we assume that the deviations are entirely due to this cause.

Comparison of the figures in column 4 shows at a glance the "ironing out" effect of the pillars on the character of the decay of sound below 512 cycles. The data show no unmistakable effect at the two higher frequencies, indicating that at these frequencies instrumental errors rather than fluctuations in the decay are largely responsible for the deviations. The much steeper line of the decay at the high frequencies tends to exaggerate the effect of these errors as measured by the criterion we have chosen. Indicating the improvements, may be noted the fact that a careful determination of the decay curve at 128~ before the columns were installed showed a maximum deviation of 0.11 seconds from the logarithmic law. After the installation the maximum was 0.022 seconds.

TABLE I.

Empty Room

	Without Columns				With Columns			
f	1	2	3	4	1	2	3	4
137	7.0	0.362	0.097	0.68	9.9	0.148	0.039	0.39
256	7.5	.246	.073	.55	9.0	.192	.056	.50
512	7.3	.186	.059	.43	8.9	.132	.039	.34
1024	7.6	.137	.043	.33	9.4	.099	.032	.30
2048	11.1	.082	.029	.32	13.6	.076	.023	.31
With 72 sq. ft. Absorbent Material								
137	7.83	—	.138	1.08	10.9	.115	.035	.43
256	9.20	.195	.059	.54	10.7	.177	.035	.38
512	12.84	.092	.038	.49	14.7	.083	.029	.43
1024	14.80	.077	.030	.44	16.4	.068	.022	.36
2048	16.68	.064	.018	.30	19.7	.047	.022	.42

TABLE II. *Influence of pillars on area effect.*

Frequency	Without	With	Without	With	Without	With	Without	With
Sample Size	256		512		1024		2048	
5× 5 ft.	0.49	0.16	0.84	0.76	1.01	0.98	1.01	0.89
6× 6 "	.50	.35	.77	.81	.93	.97	.92	.88
7× 7 "	.45	.33	.71	.76	.91	.93	.91	.89
8× 8 "	.46	.33	.71	.75	.88	.93	.92	.84
9× 9 "	.43	.36	.66	.72	.88	.87	.87	.83
10×10 "	.42	.30	.67	.73	.84	.84	.82	.82

Area Effect

It is a well-established fact that a small area sample of an absorbent material gives higher values of the absorption coefficient as measured by the reverberation method. This results from the fact that with localized absorption we have not a diffuse distribution but one in which on the average there is a greater flow of energy toward the absorbent area than towards an equal nonabsorbent area. This is true regardless of whether we consider the decay of sound from the statistical point of view or as the free vibrations of a three-dimensional body of air. Therefore it is not to be expected that the presence of the pillars will of necessity have any influence upon this so-called area effect. The results of experiments on this point are shown in Table II. Rates of decay over a 40 db range with and without the pillars present were determined when different areas ranging from 25 to 100 sq. ft. of a highly absorbent material were introduced. The equivalent absorptions were computed from the difference in the rates of decay with and without absorbent material. The apparent coefficients were obtained by dividing the increase in equivalent absorption by the area of the sample present.

Inspection of the table suggests no general statement that will cover all the experimental results. Without the pillars, the general tendency

TABLE III.

	Without Pillars		With Pillars	
	Sample in Position 1	Sample in Position 2	Sample in Position 1	Sample in Position 2
128	10.6 db/sec.	10.9 db/sec.	14.2 db/sec.	14.2 db/sec.
256	14.1 "	14.6 "	16.7 "	16.7 "
512	14.5 "	13.4 "	16.6 "	16.7 "
1024	15.7 "	15.4 "	17.1 "	17.1 "

of decreasing measured coefficients with increasing sample size is apparent at all frequencies. At the two lower frequencies the influence of the pillars in reducing this tendency is unmistakable. With two exceptions the variation in the measured coefficients is little if any greater when the pillars are in the room than would be obtained in making five independent measurements on a fixed sample size. At 1024~, the values with and without the pillars agree within the range of the normal deviations in measurement. At 2048~ the precision of measurement is much less than at lower frequencies due to the higher rate of decay. Thus a variation of 0.02 second in determining the time of a 40 decibel decay would effect a variation of 10 percent of the value of the measured coefficient of the 5×5 ft. sample. Hence it is quite probable that the presence of the pillars do not effect the variation of measured coefficient with sample size at this frequency.

Effect of Sample Position

It had been observed that with the reverberation chamber in its original condition, the effect of a very highly absorbent sample on the rate of decay depended to a slight degree upon its position in the room. With materials of moderately high coefficients this effect was not appreciable. Due to the presence of the organ used in earlier measurements which was placed in one corner of the room there was a degree of dissymmetry. It was thought that possibly this lack of symmetry might account for the variation caused by change of sample position. Tests were accordingly conducted to determine what effect if any would result from the presence of the pillars. A test sample 8×9 feet and $4\frac{1}{4}$ inches thick was used. Very careful measurements were made of the rate of decay at the four lower frequencies with the absorbent material in positions 1 and 2, as shown in Fig. 1, before and after the pillars were introduced. The results shown in Table III show that with the pillars in the room the rate of decay is independent of the sample position.

Effect on Measured Coefficients

Finally it is important to know what effect if any, upon the measured values of the coefficients of materials results from the modification of test conditions by introducing the pillars into the sound chamber. Accordingly, the results of routine measurements made both before and after the pillars were brought in on each of four different samples covering a wide range of absorptivities were compared.

The values averaged for the four materials shown below indicate that the presence of the pillars produce no important difference in the results of measurements.

	Coefficients				
Frequency	128	256	512	1024	2048
Before	0.25	0.48	0.74	0.89	0.80
After	0.27	0.46	0.77	0.90	0.80

Summarizing the results of the experiment it may be stated that at frequencies below 512 cycles, the diffusing effect of the pillars operates in a measurable degree in the direction of a more nearly diffuse distribution of the sound during the decay period, without producing any essential difference in measured values of absorption coefficients of materials. It is thus a distinct advantage in the measurement of coefficients at the two lower frequencies. At 512 and 1024 cycles, the effect either way of the pillars on test conditions is negligible. At 2048 cycles the precision of measurements is somewhat decreased by the fact that the pillars increase the rate of decay for the empty room, thus enhancing the effect of slight deviations in timing upon the computed value of the coefficients. On the whole, the advantages in favor of this modification seem to warrant its retention as a permanent part of the test conditions for absorption measurements.

The Diffraction Produced by Cylindrical and Cubical Obstacles and by Circular and Square Plates

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THE diffraction of a sound field by an obstacle has for many years been the subject of academic discussion among those interested in acoustics and in more recent years it has been recognized as of considerable practical importance.

Efforts to derive theoretical expressions representing the effects of diffraction by obstacles of practical size have met with very little success except in the case of a spherical object. This problem relating to the sphere was first successfully solved by Lord Rayleigh.[1] Theories applying to semi-infinite planes,[2] very long cylinders and paraboloids[3] of infinite extent have also been derived but they yield very little information regarding the effects of obstacles of practical dimensions.

During recent years a considerable amount of attention has been given at Bell Telephone Laboratories to the diffraction caused by square and circular baffles. The results of some of this work have been published by Sivian and O'Neil.[2] The present paper presents information covering wider ranges of the variables and, in addition, considers the diffraction effects of cylinders and cubes.

Complete general mathematical expressions for the diffraction caused by obstacles of these shapes have been derived but we have been unable to evaluate the arbitrary constants so as to satisfy all of the boundary conditions involved. These general solutions are, however, being presented in this paper for whatever value they may be to other investigators.

Approximate solutions have been derived which express with considerable accuracy the pressure at any point on one face of the obstacle when that point is directly in line with an oncoming plane sound wave. In arriving at these solutions it was assumed that the sound reflected from each infinitesimal element of area on this face of the obstacle is equal to that which would be radiated by a similar element vibrating in an infinite baffle. The velocity of this vibration was assumed to be equal and opposite to the particle velocity that the incident wave would cause in free space in a direction normal to the element of area. It was also assumed that the effects of reflections from the other surfaces of the obstacle may be neglected in comparison with those arising from reflections at the surface under consideration.

Numerical values of the ratio of the pressure at the center of one face of the obstacle to that in the unobstructed sound wave were determined from the approximate solutions and are shown in curve form together with corresponding experimental data. These ratios are measures of the effects of diffraction in the immediate vicinity of the obstacle.

To test the accuracy of the computed curves, measurements were made of the change in pressure caused by inserting cylinders and cubes in a sound field. The cylinder chosen for this test was six inches in diameter and six inches long. The edges of the cubical obstacle were also six inches in length. Obstacles of these dimensions were chosen because they caused several maxima and minima to occur within the audiofrequency range. To throw additional light on the validity of the assumptions made in deriving the equations employed in computing the diffraction, a six-inch circular baffle and a six-inch square baffle were included in these tests. As in the case of the cylinder and the cube, provision was made for mounting a small search microphone at the center of both baffles. In order that the wave front be approximately plane when it reached the obstacle the sound source was located at least four feet from the nearest face. Certain considerations involved in the measurements made it impractical to separate the sound source and

[1] Rayleigh, *Theory of Sound*, Vol. 2, p. 2, Papers No. 287; Vol. V, p. 112 and No. 292; Vol. V, p. 149.
[2] L. J. Sivian and H. T. O'Neil, "On Sound Diffraction Caused by Rigid Circular Plate, Square Plate and Semi-Infinite Screen," J. Acous. Soc. 3, 483–510 (1932).
[3] M. Y. Rocard, "Les Paraboloides Acoustiques," Rev. d'acoustique, Juillet 222 (1932).

obstacle by a greater distance. A number of sound sources were tried during the course of this work, but an artificial mouth[4] and a high frequency loudspeaker[5] were found to be most satisfactory. The artificial mouth was used for frequencies below 3500 cycles per second and the high frequency loudspeaker for the frequencies up to 15,000 cycles per second. There was, of course, no sharp line of demarcation in the use of these sources and in each instance there was sufficient overlapping of the observed points to tie in the data taken with the two types.

To reduce the effects of reflections from walls and other objects, all of the pressure measurements were made out of doors with the sound source and the obstacle under test mounted on a light weight supporting structure more than 10 feet above the top platform of a 50-foot tower. As an additional precaution the surfaces of the platform and supporting structures were covered with hair felt. The measuring equipment was located in a building at the base of the tower.

The ratio of the pressure at the center of one surface of the obstacle to the pressure at the same point in space after the obstacle had been removed was determined for various frequencies and angles of incidence by observing the change in the reading of the measuring equipment which was calibrated in terms of decibels. A high pass filter having a 120 c.p.s. cut-off was introduced in the measuring circuit to reduce low frequency interference, and a variable low pass filter was employed to reduce the effects of harmonics.

Although making these measurements out of doors aided materially in reducing reflection from surrounding objects, other problems were introduced which required consideration. One of the most important of these was the effect of wind. During most of each day there were air currents of sufficient magnitude to affect seriously the precision of the readings; therefore, most of the observations were made in the early morning when the intensity of these currents was a minimum.

[4] A. H. Inglis, C. H. G. Gray and R. T. Jenkins, "A Voice and Ear for Telephone Measurements," Bell System Tech. J., April, 295–317 (1932).

[5] L. G. Bostwick, "An Efficient Loud Speaker at the Higher Audible Frequencies," J. Acous. Soc. 2, 242 (1930).

General Solutions

In obtaining completely general and exact equations representing the effects of obstacles on a sound field it is necessary to secure solutions for the following differential equation governing the propagation of sound waves in a homogeneous medium[6]

$$\nabla^2\Phi+k^2\Phi=0. \tag{1}$$

In this equation Φ represents either the pressure or the velocity potential (which are proportional to each other) at any point in space and k is equal to 2π divided by the wave-length, λ, of the sound involved.

Cylindrical obstacle

In the case of the right circular cylinder, cylindrical coordinates are the most convenient. The differential equation then assumes the form

$$\frac{\partial^2\Phi}{\partial r^2}+\frac{1}{r}\frac{\partial\Phi}{\partial r}+\frac{1}{r^2}\frac{\partial^2\Phi}{\partial\Phi^2}+\frac{\partial^2\Phi}{\partial z^2}+k^2\Phi=0. \tag{2}$$

A complete formal solution of this equation is

$$\Phi=\frac{A_0\exp\left(ik(r^2+z^2)^{\frac{1}{2}}\right)}{(r^2+z^2)^{\frac{1}{2}}}+\frac{B_0\exp\left(-ik(r^2+z^2)^{\frac{1}{2}}\right)}{(r^2+z^2)^{\frac{1}{2}}}$$

$$+\sum_{n=0}^{\infty}\left[C_n\epsilon^{in\theta}+D_n\epsilon^{-in\theta}\right]\left[E_n\epsilon^{cz}+F_n\epsilon^{-cz}\right]$$

$$\times\left[G_nJ_n(ar)+H_nK_n(ar)\right], \tag{3}$$

where n is a whole number and c is equal to $(a^2-k^2)^{\frac{1}{2}}$. There is no limitation on the value of a and hence none on c. Each of these constants may be real, imaginary or complex and may assume an infinite number of values for each value of n. $J_n(ar)$ and $K_n(ar)$ represent the nth order Bessel functions of the first and second kinds, respectively. In the general solution, the coefficients A, B, etc., are completely arbitrary and in a particular solution they are controlled by the boundary conditions which also require that n be a whole number.

In the case of the circular cylinder there are four boundary conditions, three of which are due to the fact that the particle velocity normal to each of the three surfaces of the cylinder must

[6] Rayleigh, *Theory of Sound*, Vol. 2, p. 69.

be equal to zero. The fourth boundary condition arises from the fact that the reflection from the obstacle causes the same amount of reflected sound energy to pass across all hypothetical surfaces surrounding the obstacle.

Cubical obstacle

In the case of the cubical obstacle rectangular coordinates are best suited to the problem. The differential equation then assumes the form

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}+k^2\Phi=0. \qquad (4)$$

A complete formal solution of this equation is

$$\Phi=\frac{A_0\exp\,(ik(x^2+y^2+z^2)^{\frac{1}{2}})}{(x^2+y^2+z^2)^{\frac{1}{2}}}+\frac{B_0\exp\,(-ik(x^2+y^2+z^2)^{\frac{1}{2}})}{(x^2+y^2+z^2)^{\frac{1}{2}}}+\Sigma[A_m\epsilon^{(a_m+ib_m)x}+B_m\epsilon^{-(a_m+ib_m)x}]\times[C_m\epsilon^{(c_m+id_m)y}+D_m\epsilon^{-(c_m+id_m)y}]\times[E_m\epsilon^{(g_m+ih_m)z}+F_m\epsilon^{-(g_m+ih_m)z}]. \qquad (5)$$

The constants involved in the exponents are practically arbitrary, the only limitations on them being

$$a_m^2-b_m^2+c_m^2-d_m^2+g_m^2-h_m^2=-k^2,$$

and

$$a_mb_m+c_md_m+g_mh_m=0.$$

The coefficients, A_m, B_m, etc., are completely arbitrary and are controlled by the boundary conditions.

There are seven boundary conditions, six of which are due to the fact that the particle velocity normal to each of the six faces must be equal to zero. As in the case of the cylinder the other boundary condition arises from the fact that the same amount of reflected energy must pass across all surfaces surrounding the cube.

Although several methods were tried, every attempt to satisfy simultaneously all of the boundary conditions imposed in the general solution of the effects of either the cylindrical or the cubical obstacle on a sound field was unsuccessful.

APPROXIMATE SOLUTIONS

The total velocity potential, Φ, at any point on the surface of the obstacle may be considered equal to the sum of the potential, Φ_0, due to the incident sound wave and a secondary potential, Φ_s, due to an assumed virtual vibration of the surface. This vibration is equal and opposite to the normal component of the particle velocity which would be produced in free space by the incident wave. It is a mathematical device which makes it possible to satisfy the boundary condition that the particle velocity, normal to the surface of the obstacle must be equal to zero.

In deriving the approximate equations, it was assumed that the virtual vibration of each infinitesimal element of area produced an effect equal to a similar elemental sound source vibrating in an infinite baffle, and that the effects of the virtual vibrations of the other surfaces of the obstacle could be neglected in comparison with those of the surface under consideration. The total secondary velocity potential, Φ_s, was then determined by integrating the effects of the virtual vibrations within the boundaries of the surface. These assumptions are the same as those made by Sivian and O'Neil[2] in the derivation of the equations presented in their paper.

For convenience, the origin of coordinates was placed at the point for which the pressure was to be determined. Moreover, it was assumed that the surface was in a vertical plane; that the sound wave approached in a horizontal direction and had a plane wave front of undiminishing amplitude; and that in the case of the square surface two of the edges were horizontal.

Let φ be the angle between the direction of approach of the plane sound wave and a normal to the surface under consideration. As the sound wave advances toward the surface a wave crest will arrive at one side of the surface before it reaches the other. Hence, the phase shifts from point to point unless φ is equal to zero. As indicated by Fig. 1 which represents a horizontal projection of the surface and the oncoming sound wave, the crest of sound pressure at P follows that at the origin by an interval of time equal to $d/c=a/c \sin \varphi$, where c is the velocity of sound. If r is the radius vector on the surface from the origin to the point P, a is equal to the horizontal component of this vector or $r \cos \theta$,

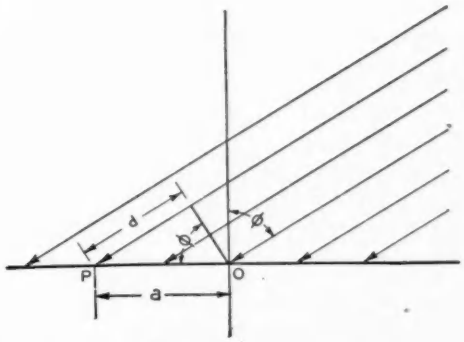

FIG. 1. Horizontal projection representing the sound wave and a surface of the obstacle.

θ being the angle which the radius vector makes with the horizontal reference line. Therefore, if the velocity potential at any point on the surface be expressed with respect to that at the origin,

$$\Phi_P = \Phi_0 \epsilon^{-i\omega(t+r(\cos\theta \sin\varphi)/c)}$$
$$= \Phi_0 \epsilon^{-i(\omega t + kr\cos\theta\sin\varphi)}. \quad (6)$$

As the incident sound wave advances in free space the velocity potential at any point is defined by the equation

$$\Phi_p = \Phi_0 \epsilon^{-i(\omega t + kq)}, \quad (7)$$

where q is a measure of distance in the direction of propagation. The particle velocity in this direction is

$$v = -\partial\Phi_p/\partial q = ik\Phi_p, \quad (8)$$

and its component in a direction normal to the surface under consideration is

$$v_n = ik\Phi_p \cos\varphi.$$

Therefore, in order to meet the boundary condition that the actual velocity normal to the surface shall equal zero, the assumed virtual velocity must be

$$u_P = -v_n = -ik\Phi_P \cos\varphi. \quad (9)$$

Φ_p applies anywhere in space. Φ_P applies only at the obstacle surface and, therefore, is a special case for Φ_p. Then from Eqs. (6) and (9),

$$u_P = -ik\Phi_0 \cos\varphi \epsilon^{-i(\omega t + kr\cos\theta\sin\varphi)}. \quad (10)$$

Neglecting the harmonic time factor this equation becomes

$$u_P = -ik\Phi_0 \cos\varphi \epsilon^{-ikr\cos\theta\sin\varphi}. \quad (11)$$

The increment of secondary velocity potential at the origin due to the virtual vibration of an infinitesimal area $r\,d\theta\,dr$ is equal to

$$d\Phi_s = \frac{u_P r\,d\theta\,dr}{2\pi r} \cdot \epsilon^{ikr}, \quad (12)$$

ϵ^{ikr} representing the phase difference due to travel of the impulse from the small area to the origin.

Therefore, the total secondary potential at the origin is

$$\Phi_s = -\frac{ik\Phi_0\cos\varphi}{2\pi}\int_0^{2\pi}\int_0^{\rho}\epsilon^{ikr(1-\sin\varphi\cos\theta)}dr\,d\theta. \quad (13)$$

In this equation ρ is the value of r at the boundary of the surface.

As previously stated, the total pressure at the origin is proportional to $\Phi_0+\Phi_s$. Therefore, the ratio of the total pressure to that in the free sound field is

$$\frac{P}{P_0} = \frac{\Phi_0+\Phi_s}{\Phi_0} = 1 - \frac{ik\cos\varphi}{2\pi} \times \int_0^{2\pi}\int_0^{\rho}\epsilon^{ikr(1-\sin\varphi\cos\theta)}dr\,d\theta. \quad (14)$$

Although Eqs. (13) and (14) are valid for any point on any plane surface, their use was limited in this paper to points at the centers of the square and circular surfaces of the obstacles under consideration.

Obstacle with circular bounding surface

In the case of the plane circular surface ρ becomes a constant R when the origin of coordinates is at the center of the surface. Substituting this limit in Eq. (13) and integrating with respect to r, there results

$$\Phi_s = -\frac{\Phi_0\cos\varphi}{2\pi}\epsilon^{ikR}\int_0^{2\pi}\frac{\epsilon^{-ikRp\cos\theta}d\theta}{1-p\cos\theta} + \frac{\Phi_0\cos\varphi}{2\pi}\int_0^{2\pi}\frac{d\theta}{1-p\cos\theta}, \quad (15)$$

p being equal to $\sin\varphi$.

The term $\frac{\Phi_0 \cos\varphi}{2\pi}\int_0^{2\pi}\frac{d\theta}{1-p\cos\theta}$ reduces to[7] $(\Phi_0 \cos\varphi)/\pm(1-p^2)^{\frac{1}{2}}$ which is equal to $\pm\Phi_0$. Therefore, Eq. (14) becomes

$$\frac{P}{P_0}=1\pm 1-\frac{\epsilon^{ikR}\cos\varphi}{2\pi}\int_0^{2\pi}\frac{\epsilon^{-ikRp\cos\theta}d\theta}{1-p\cos\theta}. \quad (16)$$

From a physical consideration of the problem it is obvious that the positive sign should be used for the second term of this equation for all angles of incidence less than 90°, inasmuch as it is known that under these conditions there tends to be an increase of the pressure at the surface of the obstacle. Since the obstacle acts as a shield when the angle of incidence is between 90° and 180°, the negative sign should be used for that range of values. For the special case where the angle of incidence is equal to 90° the pressure at any point on the surface is equal to the incident pressure. The value of p is equal to 1 at this angle and the equations given in this section are not valid for that value.

For purposes of numerical calculation the numerator and the reciprocal of the denominator of the integral, I, of Eq. (16) were expanded individually in series. Then

$$I=-\frac{\epsilon^{ikR}\cos\varphi}{2\pi}\int_0^{2\pi}\left(1-ikRp\cos\theta-\frac{k^2R^2p^2\cos^2\theta}{2!}+\frac{ik^3R^3p^3\cos^3\theta}{3!}+\cdots\right)$$

$$\times(1+p\cos\theta+p^2\cos^2\theta+p^3\cos^3\theta+\cdots)d\theta. \quad (17)$$

The first of these series converges for all values of $kRp\cos\theta$ and the second converges for all cases where the absolute value of p is less than 1. Since $p=\sin\varphi$ values of p greater than 1 are of no interest in connection with this problem. Multiplying and integrating term by term, Eq. (16), which represents the diffraction effect on the circular surfaces of certain obstacles, then becomes

$$I=-\epsilon^{ikR}\cos\varphi\left[\left(1+\frac{p^2}{2}+\frac{3p^4}{2\times4}+\frac{3\times5p^6}{2\times4\times6}+\cdots\right)\right.$$

$$-\left(ikR+\frac{k^2R^2}{2!}\right)\left(\frac{p^2}{2}+\frac{3p^4}{2\times4}+\frac{3\times5p^6}{2\times4\times6}+\cdots\right)$$

$$+\left(\frac{ik^3R^3}{3!}+\frac{k^4R^4}{4!}\right)\left(\frac{3p^4}{2\times4}+\frac{3\times5p^6}{2\times4\times6}\right.$$

$$\left.+\frac{3\times5\times7p^8}{2\times4\times6\times8}+\cdots\right)$$

$$+\ \cdots\cdots\cdots\cdots$$

$$\left.+\ \cdots\cdots\cdots\cdots\right]. \quad (18)$$

Obstacle with square bounding surface

In the case of the square surface it may appear that it would be best to utilize rectangular coordinates in evaluating the integral of Eq. (14). This, however, did not prove to be true. After several unsuccessful attempts to evaluate the integral by other methods, it was found possible to obtain a solution by using polar coordinates and integrating successively over the four subdivisions formed by drawing the diagonals of the square. Then, the total integral breaks up into four parts, and from Eq. (14)

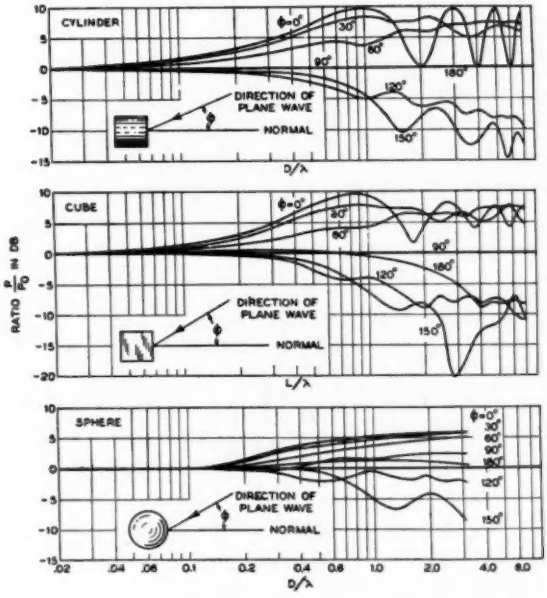

FIG. 2. Ratio of the pressure on a rigid obstacle to the pressure in the incident sound wave.

[7] D. Bierens de Haan, *Nouvelles Tables D'Integrales Definies* (1867); Eq. (12), Table 64, p. 99.

$$\frac{P}{P_0}=1-\frac{ik\cos\varphi}{2\pi}\left[\int_{-\pi/4}^{\pi/4}\int_0^{S\sec\theta}\epsilon^{ikr(1-p\cos\theta)}dr\,d\theta+\int_{\pi/4}^{3\pi/4}\int_0^{S\csc\theta}\epsilon^{ikr(1-p\cos\theta)}dr\,d\theta+\int_{3\pi/4}^{5\pi/4}\int_0^{-S\sec\theta}\epsilon^{ikr(1-p\cos\theta)}dr\,d\theta+\int_{5\pi/4}^{7\pi/4}\int_0^{-S\csc\theta}\epsilon^{ikr(1-p\cos\theta)}dr\,d\theta\right], \quad (19)$$

where S is half the length of the edge of the square. Integrating with respect to r and making appropriate changes in the remaining variable of some of the resulting integrals, Eq. (19) can be put in the form,

$$\frac{P}{P_0}=1-\frac{\cos\varphi}{\pi}\left[\int_0^{\pi/4}\frac{\epsilon^{ikS\sec\theta(1-p\cos\theta)}d\theta}{1-p\cos\theta}+\int_0^{\pi/4}\frac{\epsilon^{ikS\sec\theta(1+p\cos\theta)}d\theta}{1+p\cos\theta}+\int_{-\pi/4}^{\pi/4}\frac{\epsilon^{ikS\sec\theta(1+p\sin\theta)}d\theta}{1+p\sin\theta}\right]+\frac{\cos\varphi}{2\pi}\int_0^{2\pi}\frac{d\theta}{1-p\cos\ell}. \quad (20)$$

As stated above, the last of these integrals reduces to ± 1. An attempt was made to evaluate the other three integrals analytically. No simple expressions were obtainable for them, however, and when series were resorted to, it was found that they were either divergent over the whole range of values covered in this paper or divergent over part of the range and too slowly convergent over the remainder to be of any practical use. The following series is an example:

$$\frac{P}{P_0}=1\pm 1-\frac{\cos\varphi}{\pi}\left(\frac{\pi}{\cos\varphi}-\frac{4k^2S^2}{2!}+\frac{4(4+6p^2)k^4S^4}{3\times 4!}-\frac{28(4+20p^2)k^6S^6}{3\times 5\times 6!}-\frac{18p^4k^6S^6}{6!}-\cdots\right.$$
$$\left.+1\left[4\times 0.881kS-\frac{2.295(4+2p^2)k^3S^3}{2\times 3!}+\frac{12.541(4+12p^2)k^5S^5}{2\times 4\times 5!}\right]\right). \quad (21)$$

This was obtained by expanding the numerators of Eq. (20) by means of Taylor's series and integrating term by term. For the number of terms calculated, this series has been found useful for value of kS up to 3. For values exceeding 3, it was found that a much greater number of terms would be required to secure the desired accuracy and that the work involved in integrating to obtain these additional terms would be prohibitive.

It was felt, therefore, that a graphical method of integrating was the most appropriate. Hence, in obtaining the theoretical curves presented herein, the integrands were plotted for the desired values of kS and p and the areas under the resulting curves determined by means of a planimeter. The results so obtained checked very well with those resulting from substitution in Eq. (21) for the range wherein the series could be conveniently handled.

General curves, based on these approximate solutions, are plotted on Fig. 2 for various angles of incidence. They represent the diffraction effects of a circular cylinder and of a cube as functions respectively of the radius of the cylinder and of the size of the cube. Similar curves for a spherical obstacle were calculated from a modified form of Lord Rayleigh's equation and for the sake of comparison are included on Fig. 2.

Numerical calculations of the diffraction effects of a six-inch cylinder, a six-inch circular plate, a six-inch cube and a six-inch square plate are shown by the theoretical curves on Figs. 3 and 4. As previously implied, it was assumed that the same equations would apply to the cylinder as to the circular plate and that a second set of equations would apply as well to the cube as to the square plate. In addition the experimental curves are shown.

All of these curves as well as those of Fig. 2 show the ratio (expressed in decibels) of the pressure, P, at the point of measurement on the surface of the obstacle to the pressure P_0 in the incident sound wave.

The fact that the experimentally determined curves differ so little from the theoretical curves for angles of incidence not exceeding 90° indicates that for these angles the effects of reflections from surfaces other than the one under

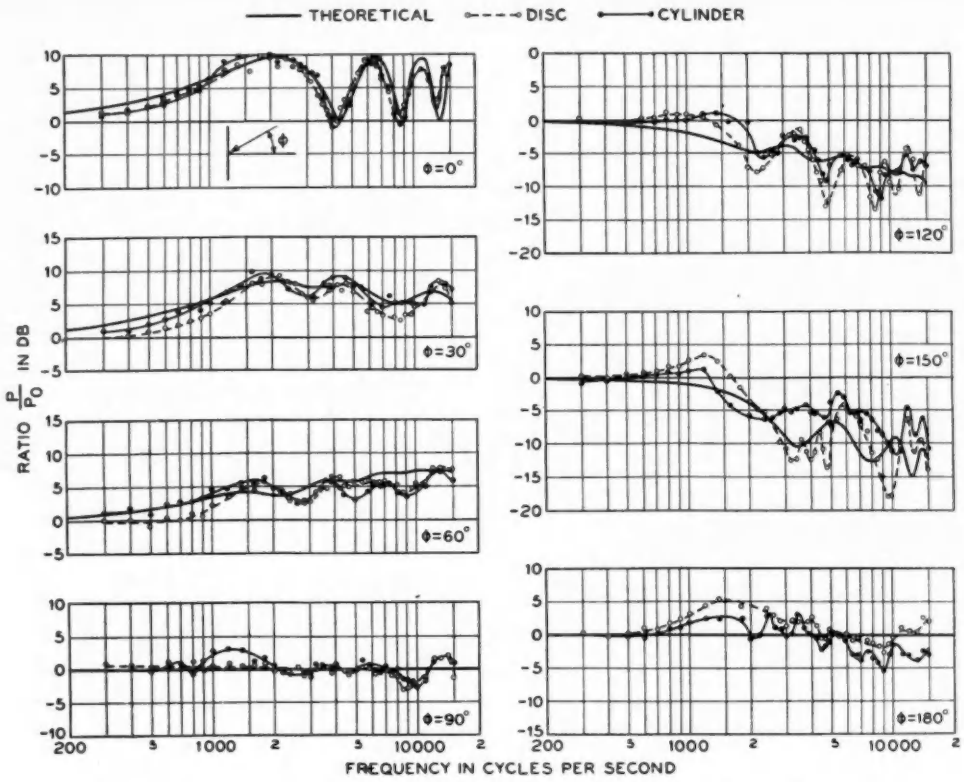

FIG. 3. Experimental and computed diffraction caused by a cylindrical obstacle.

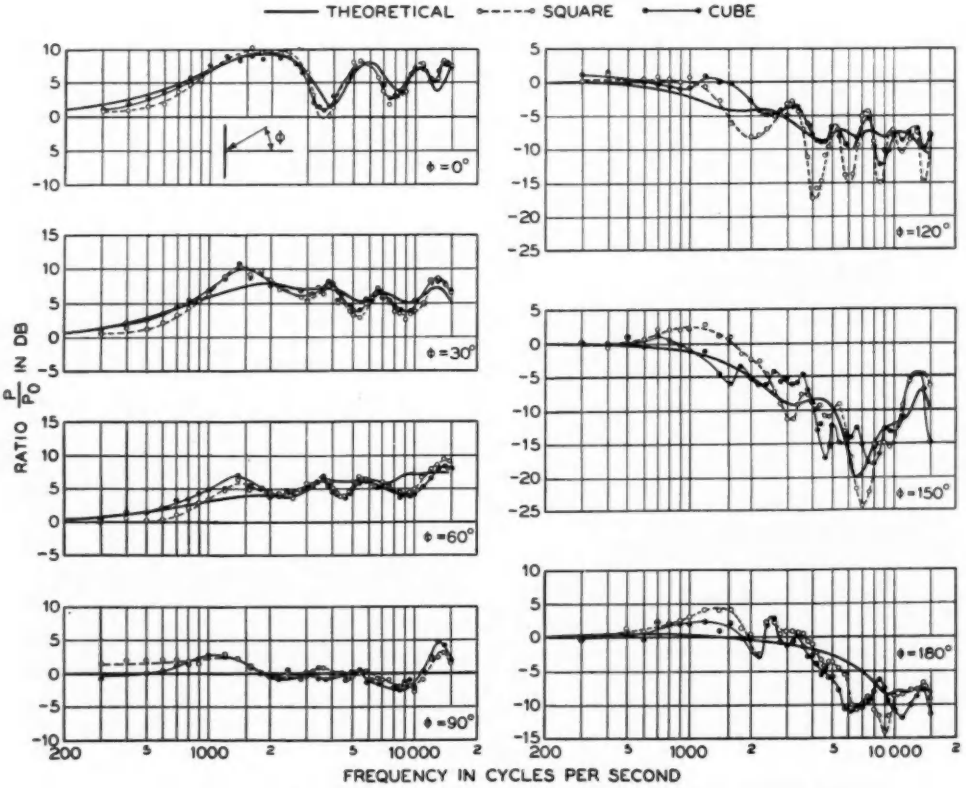

FIG. 4. Experimental and computed diffraction caused by a cubical obstacle.

consideration are unimportant and establishes the reasonableness of the assumption made in the theoretical calculations. At angles exceeding 90° the discrepancies between the theoretical and experimental curves are sufficiently large to indicate that for such angles of incidence the reflections from the other surfaces play a much more important part. Even under these conditions, however, the theoretical and experimental curves have the same general trend. This is illustrated particularly well by the very pronounced dips which occur at about 6000 c.p.s. in the 150° curves applying to the cube.

The pronounced maxima and minima in the diffraction curves for the cylinder and cube, particularly for normal incidence of the sound wave, are of interest because it is generally assumed that the diffraction of a normally incident sound wave causes a doubling of the pressure at the surface of the obstacle. This assumption probably is based on an observance of curves such as those shown on Fig. 2 for the sphere. In that case the maximum increase in pressure is 6 decibels which corresponds to doubling. On the other hand, increases in pressure as great as 10 decibels may be obtained with the obstacles considered in this paper.

Like the sphere, the pressure at the point of observation on the surface of the cylinder or the circular plate is approximately equal to that in the free sound field when the angle of incidence is 180°. This is analogous to the bright spot produced in the shadow of a circular disk by the diffraction of light.

On Distortion in Sound Reproduction from Phonograph Records

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IN the complicated process of recording and reproducing a phonograph record there are many ways in which nonlinear or harmonic distortion may enter the system. If one assumes that the electromechanical conversion is perfect in both recording and reproduction there still remain two geometric factors introducing harmonic distortion which may not be reduced except by altering the dimensions of the apparatus. The first of these is "tracking error," and may be defined as the angle between the vertical plane containing the vibration axis of the mechanical system of the reproducer and a vertical plane containing the tangent to the record groove. Such an angle arises from the convenient mechanical device of pivoting the reproducer tone arm from a fixed point. If the vibration axis of the reproducer, extended, passes through the tone arm pivot it may readily be shown that the vibration axis can never be truly tangent to the record groove at more than one value of the radius. On the other hand, if the vibration axis of the reproducer system is set at an appropriate angle with the line connecting the stylus tip and the tone arm pivot, and if the length of the tone arm is properly adjusted to the distance between the tone arm pivot and the record axis, then the tracking error may be held to a few degrees. In the case of reproduction of vertical cut records this tracking error does not introduce distortion and the tone arm may therefore be made quite short. On the other hand, in the reproduction of lateral cut records a sinusoidal groove is not traced sinusoidally by the reproducer stylus tip if the tracking error is considerable. Olney[1] has discussed this effect and has given numerical examples of the harmonic distortion which is introduced under practical recording conditions. He shows, for example, that even with so large a tracking error as 15 degrees (as large as is usually met in practice) the maximum distortion to be expected from this source is approximately 4 percent. On the other hand, it is readily possible to offset the vibration axis in such a manner that the tracking error does not exceed ± 6 degrees, and under these conditions the harmonic distortion introduced by tracking error can be neglected in comparison with other distortions met with in a practical recording system.

A very much more serious source of harmonic distortion arises from the fact that the tip of the reproducer stylus has finite size. The curve traced by the center of the tip (which we shall assume to be spherical) of a reproducer stylus sliding over a sinusoidal groove surface is not, unfortunately, sinusoidal, and it is embarrassing that the only way to reduce the distortion due to this effect is to reduce the size of the needle tip. Such reduction can only be carried so far, with the practical result that this distortion, herein called *tracing distortion*, currently remains the most serious limitation in the attainable fidelity of phonograph reproduction. DiToro[2] has discussed this type of distortion with an analysis based upon an approximate sine curve composed of parabolic and straight line sections, and has given results which are applicable to reproduction from vertical cut records. These results are satisfactory for high values of distortion, but are in some error at the lower and more interesting values. It is the purpose of this paper to present an extension of this work, based upon a method of computation which avoids DiToro's approximations, and to apply the results to an evaluation of the tracing distortion arising in the reproduction of both vertical and lateral cut records.

Harmonic Analysis of the Traced Curve

The first step in the solution of this problem lies in the harmonic analysis of the curve traced by the center of a circle which slides or rolls along a sine curve. These results will give directly the components of motion of a spherical

[1] B. Olney, Electronics **10**, 19 (1937).

[2] M. J. DiToro, J. S. M. P. E. **29**, 493 (1937).

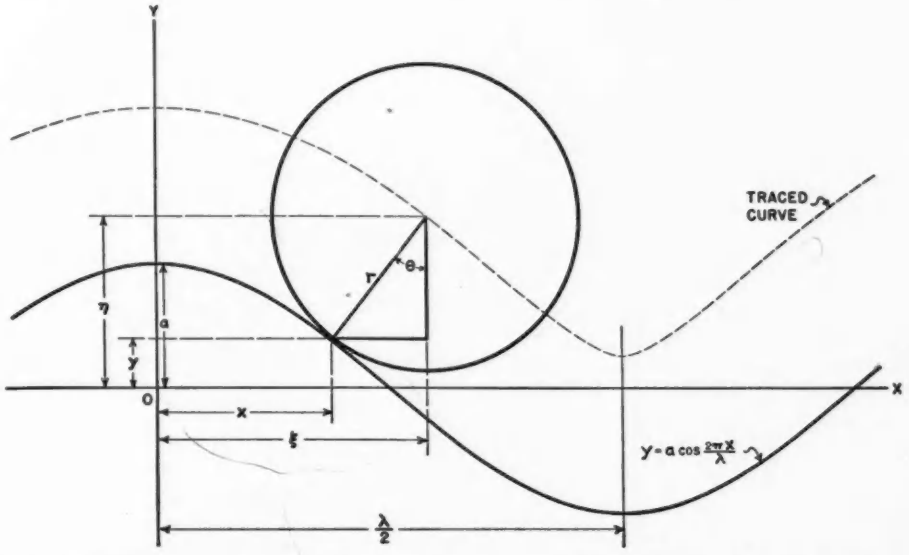

FIG. 1. Coordinates and nomenclature for the harmonic analysis of the curve ("poid") traced by the center of a circle sliding or rolling along a cosine curve.

needle tip tracing a vertically modulated record groove, and the application of the results to the analogous motion of a needle tip tracing a laterally modulated groove will be discussed later. Fig. 1 represents such a circle sliding along a cosine curve. It will be noted that the traced curve (shown dotted) is not mathematically simple. It is, however, so simple physically that it seems as though it should be numbered among the curves which are dignified by titles, and for want of a better name we have dubbed this curve the "poid" and shall so designate it in this discussion.

The coordinates of the center of the tracing circle, ξ, η (which are the coordinates of a point on the poid), may be expressed in terms of the corresponding coordinates of the point of contact of the circle with the cosine curve, as follows: Let axes be established as indicated in Fig. 1 so that the cosine curve is defined by

$$y=a\cos(2\pi x/\lambda)=a\cos kx, \qquad (1)$$

where $k=2\pi/\lambda$.

Now $\xi=x+r\sin\theta$,

but $-\tan\theta$ is the slope of the cosine curve at the point (x, y), or

$$-\tan\theta=-ka\sin kx.$$

Hence

$$\sin\theta=ka\sin kx/(1+k^2a^2\sin^2 kx)^{\frac{1}{2}}$$

and

$$\xi=x+kar\sin kx/(1+k^2a^2\sin^2 kx)^{\frac{1}{2}}. \qquad (2)$$

Similarly,

$$\eta=y+r\cos\theta$$
$$=a\cos kx+r/(1+k^2a^2\sin^2 kx)^{\frac{1}{2}}. \qquad (3)$$

These parametric equations for the poid have involved no approximations. Unfortunately the elimination of x between them and the expansion of η in a Fourier series in ξ is extremely difficult, each coefficient involving the term-by-term integration of the product of three infinite series, none of which converges rapidly for the range of values in which we are interested.

An alternative method of solution was therefore sought. Chaffee[3] has described a simplified form of schedule analysis, applicable to even functions, which is capable of considerable accuracy and requires a knowledge of the coordinates of only a limited number of points. The poid is an even function, as indicated by its symmetry about either a maximum or a minimum point, so that, with the coordinates of only seven points in a half-wave-length, the amplitudes of the second and third harmonics

[3] E. L. Chaffee, Rev. Sci. Inst. 7, 384 (1936).

may be determined within one or two percent, and reasonably accurate values may be calculated for harmonics up to the sixth. This precision was deemed adequate for our purpose.

It was required, then, to determine values of η corresponding to the values of $k\xi$ prescribed by the harmonic analysis schedule. The values of x corresponding to these prescribed values of $k\xi$ were first obtained by successive approximations, assumed values of x being inserted in Eq. (2) until $k\xi$ was established with sufficient accuracy. These values of x for each prescribed point were successively inserted in Eq. (3) and the resulting values of η used to enter the harmonic analysis schedule. The relative amplitudes of the six harmonics were then obtained by simple arithmetic.

This method of computation, while laborious, requires no approximations except those inherent in the schedule analysis and these may be made as small as desired by computing a sufficiently large number of points. By making check calculations using as many as thirteen points for a half-cycle it was found that the "seven-point" analysis was indeed sufficiently accurate.

On reference to Fig. 1 it may be noted that the size and shape of the poid, and therefore the amplitude of each harmonic constituent of the poid, is given by three linear dimensions, a, r, and λ. On the other hand, the *shape* of the poid and the *relative* amplitudes of its harmonic constituents are determined by the two dimensionless ratios, a/λ and r/λ. Our subsequent discussion will be simplified if we take these ratios as $2\pi a/\lambda$ and $2\pi r/\lambda$ (i.e., as ka and kr), and we may then say that the relative harmonic structure of the poid is a function of the two independent variables ka and kr. It is only necessary now to calculate the distortion corresponding to all possible values of these two variables—a straightforward but tedious process.

The values of harmonic distortion so calculated might be plotted vertically over the ka–kr plane and constitute a characteristic surface whose distance above the horizontal plane is a measure of the harmonic distortion for the condition corresponding to the coordinates ka and kr. It is frequently convenient to represent such a warped surface by projecting onto the horizontal ka–kr plane contours of selected constant values of harmonic distortion. It is thus possible to represent on a single chart the entire range of tracing distortion met under all recording conditions. The computational labor involved in obtaining such a set of contours is reduced by the preparation of a family of intermediate curves each showing harmonic distortion plotted against the variable kr with selected values of ka held constant for each of the intermediate curves. If horizontal lines are now drawn at the chosen values of the total harmonic distortion, they will cut the family of intermediate curves in a series of points which establish the pairs of coordinates for the points lying along the constant distortion contour curve. These points are then transferred to the ka–kr plane and the contours drawn as exhibited by the dashed lines of Fig. 4. By this method the harmonic analysis of some thirty poids is sufficient to establish the contour set covering the entire useful range of the independent variables ka and kr.

It may be pointed out that the harmonic analysis schedule yields the *amplitudes* of the various harmonics. Inasmuch as reproducing systems are almost invariably velocity-responsive (either intrinsically, as in the electromagnetic, or through equalization, as in the piezoelectric) each harmonic amplitude is multiplied by its harmonic number before comparison with the amplitude of the fundamental, and the contour chart of Fig. 4 is then drawn in terms of the total root-mean-square harmonic velocity distortion.[4]

As suggested above, the dashed-line contours of Fig. 4 are directly applicable to reproduction from vertical-cut records, and their interpretation under specific recording conditions will be given in a later section. Before applying these results to the reproduction of lateral cut records we must consider in greater detail the geometrical relation between a laterally modulated groove and the stylus tip. Fig. 2 is a plan view and two typical cross sections of a laterally modulated record groove. This groove is generated by a plane cutting surface which is always perpendicular to the axis of the unmodulated groove. There arises, consequently, a constriction in the width of the groove, measured perpendicular to its instantaneous direction, whenever

[4] If v_1, v_2, v_3 are the harmonic velocity components the total r.m.s. distortion is defined as

$$\text{H.D.} = (v_2^2+v_3^2+\cdots)^{\frac{1}{2}}/v_1.$$

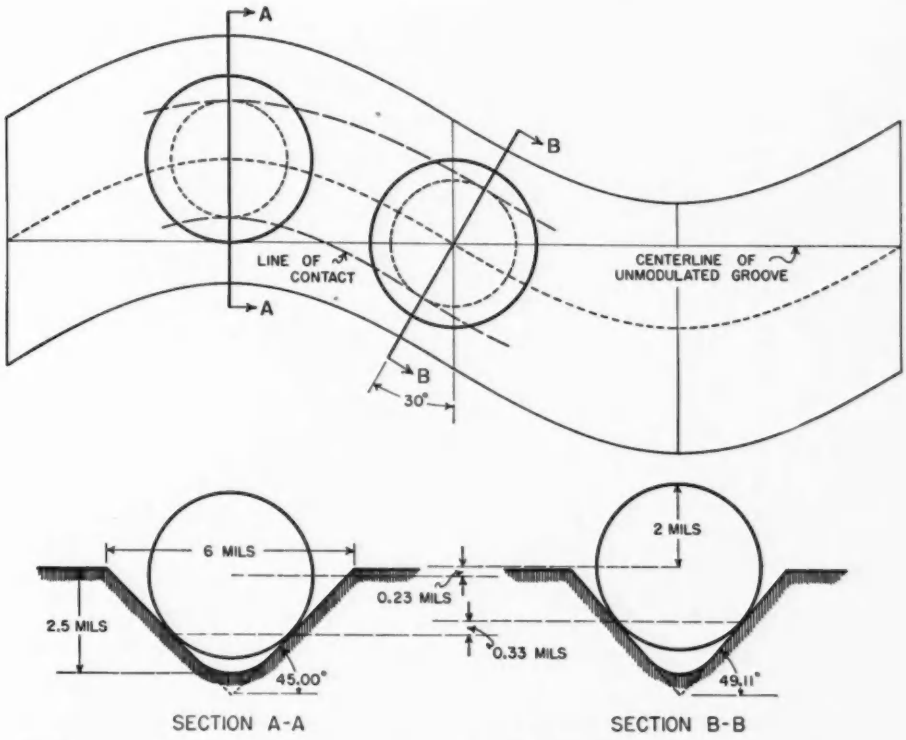

FIG. 2. Plan and sectional views of the assumed stylus bearing relations in a typical laterally modulated groove.

the cutting needle is moving at an angle to the direction of the unmodulated groove. This is illustrated by the sectional views of the groove given at the bottom of Fig. 2. If such a groove be traced by a stylus which bears at least partially on the groove sidewalls, it will be seen at once that the stylus must rise and fall twice during the tracing of each fundamental wavelength. This phenomenon appears to have been ignored or neglected in previous discussions of the so-called "pinch effect," but it leads to the necessary conclusion that *an ideal reproducer for lateral cut records must embody sufficient vertical flexibility to enable the stylus to execute this motion faithfully*. This requirement appears even more severe when it is remembered that this vertical motion must be executed at twice the frequency of the fundamental groove modulation. In typical commercial reproducers for lateral cut records there is no provision for vertical motion of the stylus relative to the tone arm, with the result that the stylus must ride at some intermediate elevation above its normal position in the unmodulated groove. Since the mass of the reproducer head and arm is too large to be vibrated at signal frequencies, the stylus is driven into the groove in the "pinched" sections. This gouges out the groove walls, producing additional surface noise and altering the original groove shape. When the groove section is not "pinched" the stylus floats above the groove and is free to "rattle" since it is not necessarily in contact with either wall. With conventional reproducing apparatus this process continues until the pinched sections of the groove have been enlarged (involving the erasure of any small amplitude high frequency modulation which may be superimposed), or the needle has been worn. The needle tip then rides at some constant level but is never thereafter positively driven by more than one groove wall at a time. Frederick[5] and Olney[1] have pointed out that distortions may arise from this condition. On the other hand, if the stylus point is sufficiently sharp to reach the rounded bottom of the "standard groove" the result is equally bad; while the tendency to vertical motion is minimized or removed, the stylus

[5] H. A. Frederick, J. S. M. P. E. **18**, 141 (1932).

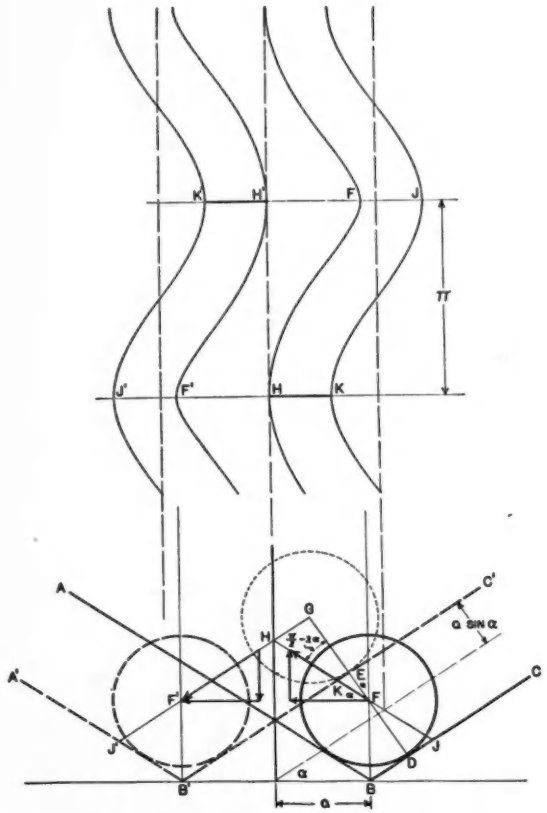

FIG. 3. Section and projected plan views illustrating the stylus displacement components during the tracing of a laterally modulated record groove.

is never positively driven by the groove. It is not possible to make an accurate quantitative analysis of the distortion introduced by the uncontrolled rattling of a stylus tip in a record groove, but it seems reasonable to assume that the distortion under such conditions will be at least as large as would be generated if the stylus were in continuous contact with the groove walls. Moreover, the distortion generated by rattling would be inharmonic, and as this would contribute noise as well as signal distortion, we may conclude that an ideal reproducer whose stylus is positively driven by the groove walls will yield a lower background noise level.

It seems, therefore, that it is highly desirable in practice, and necessary for a mathematical analysis, to endow our assumed ideal lateral reproducer with such characteristics that it can execute the prescribed vertical motion faithfully, and we further assume and hold as desirable that the stylus should be supported by the sidewalls of the groove. With a spherical stylus tip this does not lead to a large increase in needle bearing pressure in practice, and we assume here and throughout this discussion that no account need be taken of the distortion of the groove surface by the needle bearing pressure. When these conditions are satisfied we may discuss the components of stylus motion with the assistance of Fig. 3.

Consider that BC and $B'C'$ are the projections onto the plane of the paper of the two extreme positions of a radial section of one wall of a groove which is modulated laterally with an amplitude a. As the record rotates this groove wall is generated as a wavy surface having a sinusoidal profile. The curve traced by the center of a ball sliding upon this surface, if projected upon a plane perpendicular to BC, as indicated by DG, will be a poid which will have a maximum and a minimum at G and F, respectively. In this plane through DG, perpendicular to the paper, the amplitude of the sine wave is: $a \sin \alpha$, and the poid may be represented by

$$\eta' = a_1 \sin \alpha \cos k\xi + a_2 \sin \alpha \cos 2k\xi + a_3 \sin \alpha \cos 3k\xi + \cdots. \quad (4)$$

This poid may be projected upon a plane parallel to AB, such as the one through HJ, by dividing Eq. (4) by

$$\cos (\pi/2 - 2\alpha) = \sin 2\alpha = 2 \sin \alpha \cos \alpha.$$

Its equation now becomes,

$$\eta = \frac{a_1}{2 \cos \alpha} \cos k\xi + \frac{a_2}{2 \cos \alpha} \cos 2k\xi + \frac{a_3}{2 \cos \alpha} \cos 3k\xi + \cdots. \quad (5)$$

This equation expresses the motion of the sphere caused by the change in position of the sidewall represented by BC and $B'C'$, and since the motion is projected on a plane parallel to the opposite sidewall it may be divided into components, in both the vertical and horizontal planes, which are independent of any simultaneous motions of the opposite sidewall. For example, the horizontal component of the motion

is given by

$$\eta_H' = \frac{a_1}{2}\cos k\xi + \frac{a_2}{2}\cos 2k\xi + \frac{a_3}{2}\cos 3k\xi + \cdots. \quad (6)$$

The corresponding equation for the component of motion induced by the opposite sidewall is identical except for direction and a phase difference between the two poids. It is to be noted that there is a phase displacement of 180 degrees in the equation of the motion due to the sidewall indicated by AB; that is, the maximum of one poid occurs simultaneously with the minimum of the other. This is indicated by the plan view, shown in the upper part of Fig. 3, which represents the projection onto the horizontal plane of the sine curve profile of the groove walls and the poids generated in the planes through HJ and HJ'. The horizontal projection of the poid generated by the wall AB is directed oppositely to that generated by the wall BC, as viewed, say, from the center of the record, because the sphere is being pushed away from the center of the record in one case and toward it in the other. The expression for the horizontal components of motion generated by the second poid is given, therefore, by,

$$\eta_H'' = -\frac{a_1}{2}\cos(\pi + k\xi) - \frac{a_2}{2}\cos 2(\pi + k\xi) - \frac{a_3}{2}\cos 3(\pi + k\xi) - \cdots = \frac{a_1}{2}\cos k\xi - \frac{a_2}{2}\cos 2k\xi + \frac{a_3}{2}\cos 3k\xi - \cdots. \quad (7)$$

As shown by the directed arrows in Fig. 3, when the displacement of the groove is away from the center of the record the stylus is forced to move part way in this direction by the displacement of the inner groove wall, and allowed to slide the remainder of the way by the retreat of the other wall. The total lateral motion of the stylus is, therefore, the sum of the motions induced by both sidewalls, or

$$\eta_H = a_1\cos k\xi + a_3\cos 3k\xi + \cdots. \quad (8)$$

This yields at once the important result that the even harmonics of the fundamental frequency are canceled out of the lateral motion of the reproducer stylus. Returning to Eq. (5) and

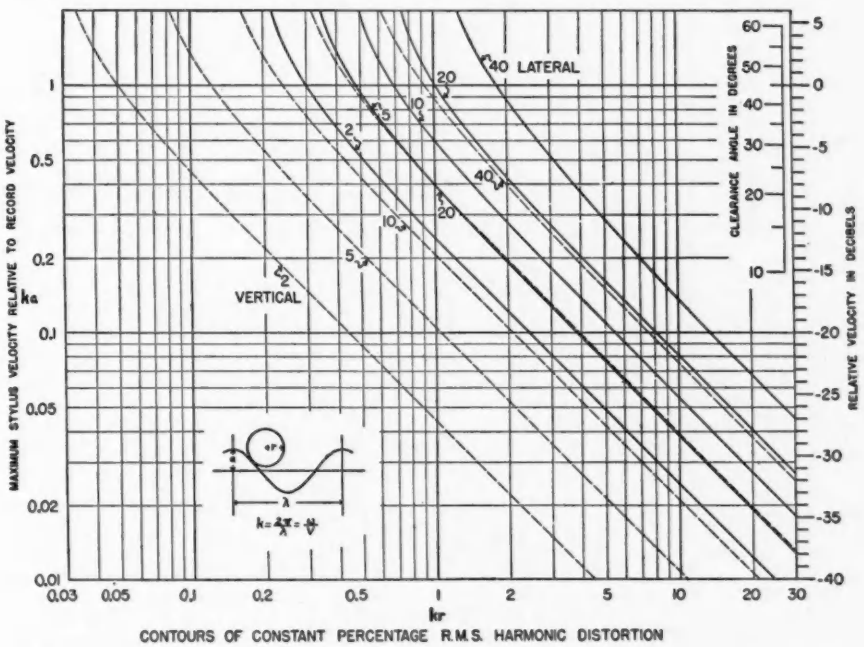

FIG. 4. Universal chart, for velocity responsive systems, displaying contours of constant r.m.s. total harmonic distortion, for both vertical and lateral cut records, plotted against the dimensionless independent variables ka and kr.

projecting the poid on the vertical plane through H by multiplying the equation by $\sin\alpha$, we find for the component of vertical motion induced by the sidewall BC,

$$\eta_V' = \frac{a_1 \tan\alpha}{2}\cos k\xi + \frac{a_2 \tan\alpha}{2}\cos 2k\xi + \frac{a_3 \tan\alpha}{2}\cos 3k\xi + \cdots. \quad (9)$$

For the component of motion induced by the opposite sidewall we have still the 180 degree displacement in phase, but the direction of the displacement represented by the poid is in this case the same, that is, upward. Hence, for the wall AB,

$$\eta_V'' = \frac{a_1 \tan\alpha}{2}\cos(\pi+k\xi) + \frac{a_2 \tan\alpha}{2}\cos 2(\pi+k\xi) + \frac{a_3 \tan\alpha}{2}\cos 3(\pi+k\xi) + \cdots$$

$$= -\frac{a_1 \tan\alpha}{2}\cos k\xi + \frac{a_2 \tan\alpha}{2}\cos 2k\xi - \frac{a_3 \tan\alpha}{2}\cos 3k\xi + \cdots. \quad (10)$$

As before, the resultant vertical motion is given by the sum of the components of motion induced by the two sidewalls and is

$$\eta_V = a_2 \tan\alpha \cos 2k\xi + \cdots. \quad (11)$$

We may, therefore, extend the conclusion stated above as follows: *the lateral motion of a stylus tracing the groove of a lateral cut record is determined by the fundamental and odd harmonics only of the poid characterized by the groove amplitude, the wave-length, and the needle radius; the even harmonics of the poid appear as vertical motion and constitute the "pinch" effect.*

It may be pointed out for contrast that in the corresponding case of a vertical cut record the motion of the two sidewalls is in phase, the maxima of the two poids occur simultaneously, the lateral motion is completely canceled out, and all of the harmonic constituents of the poid enter into the expression for the total vertical motion.

The difference in the distortions arising in the reproduction of these two types of groove modulation is emphasized by the observation that positive drive of the stylus tip by *both* sidewalls of the groove yields the usual advantages of a push-pull system, with the result that *a large part of the distortion inherent in the reproduction of vertical cut records is entirely absent in the reproduction of lateral cut records when a satisfactory lateral reproducer is employed.* The latter qualification is added to indicate that the specifications of the ideal lateral reproducer here assumed are somewhat unconventional, and these reduced distortion levels are not to be expected generally with the present types of conventional lateral reproducers.

On this basis we may return to the original data upon which the dashed contours of Fig. 4 were based and compute the root-mean-square value of the odd harmonics, prepare a set of intermediate curves, and establish the solid line contours as characteristic of the distortion generated in the reproduction of a lateral cut record.

Characteristics of the *ka–kr* Chart

We turn now to a more detailed discussion of the chart upon which contours of constant distortion are represented. A choice of logarithmic scales for the two coordinate axes endows the contour chart with the interesting property that almost all of the characteristic quantities, in terms of which the performance of a phonograph system is analyzed, are represented on the chart by straight lines which are horizontal, vertical, or inclined at 45 degrees to the principal axes.

A. The ordinate scale, ka, gives directly the ratio of the maximum cyclic stylus velocity to the tangential groove velocity, so that, other factors being held constant, a vertical line on the chart represents a change in the recorded velocity amplitude. Since the ordinate scale is logarithmic, a uniform scale, as appended to the right-hand margin of the chart, may yield the velocity amplitude in decibels referred to the tangential groove velocity as zero level. Such a scale is convenient for estimating changes in the recorded level.

The maximum radial velocity of the stylus occurs as the center line of the groove crosses the line of the unmodulated groove and the cutting angle is therefore a maximum at that time. The tangent of this angle is given directly by ka so that a subsidiary scale may be appended to the upper right-hand corner of the contour chart establishing the minimum value of the clearance angle of the cutting stylus required in order that the groove be cut without interference from the trailing edge.

B. A 45-degree line on the contour chart extending from the lower left to the upper right is a line corresponding to a constant ratio of the amplitude of groove modulation to the stylus tip radius. Along such a sloping line, drawn for a given ratio, a/r, one may plot frequency increasing upward to the right or tangential groove velocity increasing downward to the left, according as one or the other variable is assumed constant.

C. As indicated under A, a horizontal line on the contour chart is a line of constant velocity amplitude. Inasmuch as this is the usual ideal recording situation it represents an important locus, and, in general, frequency increases toward the right along such a horizontal line. There will always be, for any assumed standard conditions of record speed, maximum amplitude, and stylus radius, some record radius for which a horizontal line would allow frequency to be read directly from the scale of abscissa, in kilocycles. This record radius is frequently an unusable one but the reference line so defined is convenient in that a frequency may be located on this line and a projection made along a 45-degree line downward to the left to the record radius desired.

D. A 45-degree line extending from the lower right to the upper left is a line for which the product of ka and kr is constant. It can be shown that along such a line the ratio of the radius of the needle tip to the minimum radius of curvature of the record groove is constant. For example a line having this slope and passing through the (1, 1) point is the line for which the ratio of these curvatures is unity. All traced curves which are represented by points lying above and to the right of this line are poids having a cusp.

Approximate Generalization of the Distortion Data

One striking characteristic of the constant distortion contours exhibited in Fig. 4 is the fact that, except in the upper portion corresponding to extreme values of velocity amplitude, the contours are straight lines inclined at 45 degrees to the principal axes. Insofar as this is an accurate description of the contours we may derive certain general relationships by examining the distortion as this family of 45 degree lines is cut by other lines representing loci of constant parameters of the recording conditions.

1. As suggested above under D, a 45-degree line having the orientation of the contours is a line along which the ratio of the needle tip radius to the minimum radius of groove curvature is constant. Examination of the contours indicates that for vertical reproduction the total distortion varies linearly with the ratio of these radii. For lateral reproduction the total distortion is smaller and varies as the square of this ratio. It may be pointed out that if the frequency, the groove speed, and the needle tip radius are constant, then the ratio of the groove and needle tip curvatures, and hence the total distortion for vertical reproduction, is a linear function of the recorded amplitude. For lateral reproduction the corresponding total distortion is proportional to the square of the recorded amplitude. The ratio of the radii of curvature has frequently been offered as a criterion of the upper limit of frequency that could be reproduced satisfactorily. Our study indicates that this is indeed a satisfactory rough criterion of distortion and it may be seen from the chart that equality in these radii corresponds to approximately 40 percent total distortion for vertical reproduction and 20 percent total distortion for lateral reproduction. If one selects 10 percent total distortion as a tolerable limit the required ratios are approximately $\frac{3}{5}$ for lateral and $\frac{1}{5}$ for vertical.

The velocity amplitude of any individual harmonic component, relative to the fundamental component, may be determined by the simple empirical relationship,

$$H_n = (kakr)^{n-1}/n, \tag{12}$$

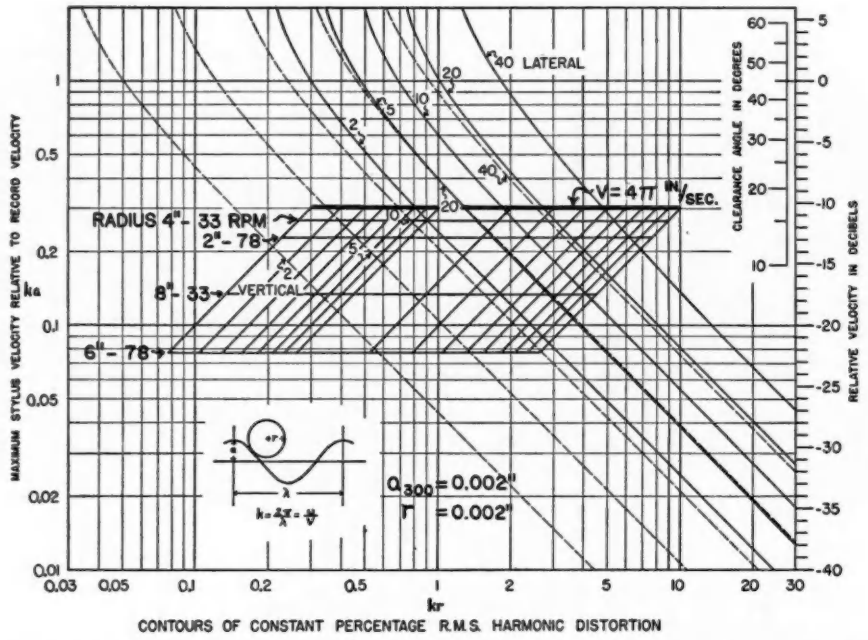

FIG. 5. Recording loci superimposed on the *ka–kr* chart. Along the heavy reference line for $V=4\pi$ in./sec. the frequency may be read directly in kilocycles from the abscissa scale.

where n is the number of the harmonic. Comparison of this relation with the preceding approximate statements about the variation of the total harmonic distortion confirms the fact that the principal component of distortion for vertical reproduction is a second harmonic, while the principal distortion component for lateral reproduction is a third harmonic.

2. A horizontal line, as discussed under C above, is a line of constant velocity amplitude, along which frequency increases towards the right. Examination of the chart reveals that total distortion varies directly with the frequency for vertical reproduction and as the square of the frequency for lateral reproduction. On the other hand, if the frequency and other reproducing conditions remain constant, then points corresponding to a variation in needle tip radius will lie along the horizontal line, and the total distortion will increase linearly with the needle tip radius for vertical and as the square of the needle tip radius for lateral.

3. If both the recorded velocity amplitude and the needle tip radius be held constant as either the record radius or the record speed is varied, the values of both *ka* and *kr* change simultaneously, so that the contour curves are crossed along a 45-degree line extending from lower left to upper right. Examination of the chart then indicates that, for vertical reproduction, the total distortion varies inversely as the square of either the record radius or the record speed according as one or the other of the two variables is held constant. For lateral reproduction the

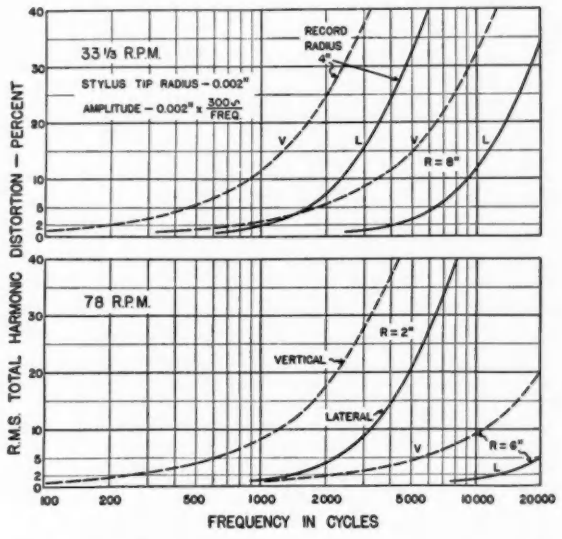

FIG. 6. Total distortion plotted as a function of frequency for constant velocity amplitude and typical recording conditions.

total distortion increases inversely as the record radius or record speed raised to the fourth power.

Summarizing these approximate relations, we may say that in general the distortion obtained in lateral reproduction is always lower than in vertical but that it varies more rapidly with the parameter introducing the distortion. Two additional characteristics of the contour chart may be pointed out in this connection. The total distortion for either lateral or vertical shows a distinct "saturation" as the distortion approaches some high value. In the case of lateral this saturation value is approximately 48 percent; for vertical the saturation value is nearly 80 percent. It will be noted that no contours have been drawn on the chart indicating the reduction in the amplitude of the fundamental component as

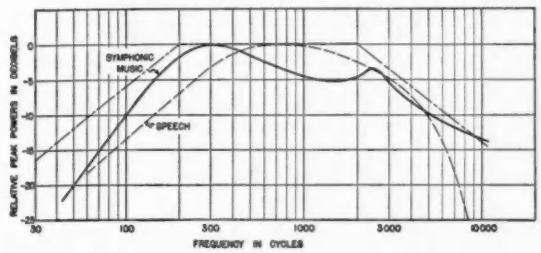

FIG. 7. Peak power as a function of frequency (from the data of Sivian, Dunn, and White) used to evaluate the distortion in recorded speech or music.

the distortion increases. This omission was made in the interests of avoiding confusion in the contour chart. The data indicate that the contour lines corresponding to 20 percent distortion for lateral or 40 percent distortion for vertical correspond roughly to a reduction in fundamental amplitude of approximately 0.7 db in either case. It is obvious therefore that the total harmonic distortion will have reached intolerable proportions before there is any significant reduction in fundamental amplitude. It follows that an appreciable "quality" difference between the inside and outside radii of a recording, detectable as a loss in high frequencies, must inevitably be accompanied by a serious increase in harmonic distortion and should never be tolerated in a high fidelity system. The provision of variable equalization for recording at the inside of a record, which has been seriously proposed, appears to be defensible only as a partial corrective for the characteristics of a recorder which relies principally upon the recording medium for damping.

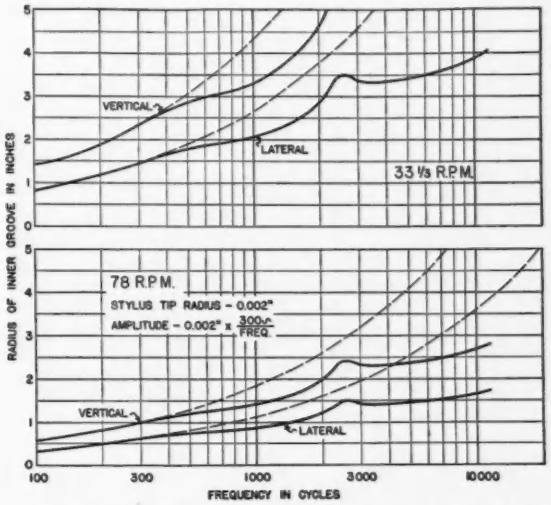

FIG. 8. Distortion data for the music spectrum (solid lines), exhibited by plotting the minimum groove radius for which the distortion will never exceed 10 percent. The dashed curves correspond to constant recorded velocity amplitude.

DISTORTION ANALYSIS FOR TYPICAL RECORDING CONDITIONS

We will now illustrate the application of the contour chart to an evaluation of the distortion arising under typical recording conditions. We will assume for this purpose that the record is cut with 100 grooves per inch, each groove being 6 mils wide at the surface of the record, 2.5 mils deep, with an included angle of 90 degrees, and having a bottom surface rounded to a radius of curvature of approximately 1.25 mils. Under these conditions the assumptions regarding support of the stylus tip by the side walls of the groove may be satisfied by a stylus having a tip radius of 2 mils. The maximum allowable amplitude of modulation, determined by the groove spacing, will also be 2 mils. If the record is vertical cut only the assumed values of a and r are material. For the standard recording conditions we will assume that the amplitude of the cut is constant for frequencies up to 300 cycles, and that the velocity amplitude is maintained constant at all higher frequencies. The constant amplitude portion of this recording locus is represented by the left-hand margin of the

parallelogram superimposed on a ka–kr chart in Fig. 5. Frequency increases upward along the left-hand border of the parallelogram and the frequency 300 cycles, at which the recording locus breaks into a horizontal line, occurs at some point dependent upon the tangential groove velocity. Four such horizontal lines are illustrated, corresponding to the inside and outside radii of typical 33 and 78 r.p.m. recordings. For the assumed stylus tip radius the reference line along which frequency may be read directly from the scale of abscissa occurs at a groove velocity of 4π inches per second. A 45-degree system of projection coordinates is based upon this reference line and allows any frequency, established along the reference line, to be referred to the appropriate record radius. For example, if it is desired to determine the distortion at 5000 cycles, 2-inch radius, 78 r.p.m., the procedure is as follows: follow the ordinate $kr=5$ upward to the standard reference line, trace downward along the 45-degree line to its intersection with the horizontal corresponding to 78 r.p.m., 2-inch radius, interpolate between the contours to determine the total (lateral) harmonic distortion as 22 percent. Following this procedure we may derive the data exhibited in Fig. 6, showing the r.m.s. total distortion at typical inside and outside radii for 33 and 78 r.p.m. recordings, both lateral and vertical cut. These curves exhibit clearly the marked superiority of lateral cut over vertical cut with regard to distortion, and they indicate also the more rapid increase in distortion for lateral cut as the frequency increases. One may be pardoned for wondering, on examination of the curves of Fig. 6, how it can be that records sound as well as they do in view of these serious distortions. The explanation of this anomaly lies in the fact that speech and music by no means present the recording medium with the necessity of recording a constant velocity amplitude at all frequencies. The valuable data of Sivian, Dunn, and White[6] on the intensity distributions in speech and music are available for an evaluation of this situation. We present our interpretation of these

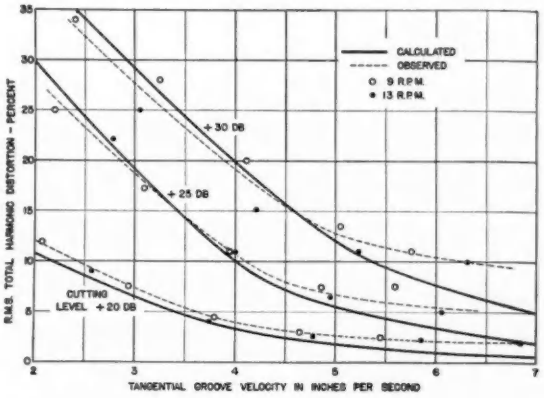

FIG. 9. Experimental data verifying the calculated distortion for lateral reproduction.

data in Fig. 7 in order to avoid any ambiguity about the application of this correction. The two curves for speech and music are arbitrarily shifted vertically to have the same peak amplitude inasmuch as an adequate volume indicator should indicate these peaks and provide a common basis for level control. The manner of applying these correction data to the distortion contours consists in locating frequencies along the broken-straight-line recording locus as before, but interpolating between the contours for a point shifted vertically downward by the number of decibels plotted for the corresponding frequency in Fig. 7. These corrected values of distortion might be shown along with the curves of Fig. 6 for constant velocity amplitude, but for the illustration of an alternative method of exhibiting these data, we have prepared the curves of Fig. 8. These show the minimum value of record radius which will allow the music spectrum of Fig. 7 to be recorded with a total distortion not exceeding 10 percent, plotted as a function of the frequency. The corresponding correction for speech would yield similar curves but without further rise beyond 2500 cycles. The nearly horizontal portions of the curves of Fig. 8 indicate that the present accepted standard inside radii for lateral cut transcription and commercial pressings are acceptable for the satisfaction of the 10 percent distortion limit, but that, for the assumed amplitude of groove modulation, the 33 r.p.m. vertical transcription record is not capable of meeting the 10 percent distortion specification without a reduction in needle tip radius.

[6] Sivian, Dunn and White, J. Acous. Soc. Am. **2**, 330 (1931). Also Fletcher, Bell System Tech. J. **10**, 349 (1931); Rev. Mod. Phys. **3**, 258 (1931); J. Acous. Soc. Am. **3** (Supp) (1931).

Experimental Confirmation of the Distortion Data

In connection with his study of vertical reproduction, DiToro[2] has measured the relative amplitude of the second harmonic for values of *ka* and *kr* lying in the lower right-hand corner of our contour diagram. The agreement between his experimental observations and our calculations is quite good for this type of measurement, and may be considered adequate to confirm the dotted contours of Fig. 4.

In view of the significantly lower distortion revealed by this study to be characteristic of lateral reproduction, it seems worth while to present some experimental data confirming these predictions. A direct reading distortion meter operating at 400 cycles was used and in order to simulate the conditions which would occur at a higher frequency at normal groove velocities, records were made and played back at speeds of 9 and 13 r.p.m. By thus shortening the wavelength, values of *ka* as high as 0.7 were obtained with *kr* no greater than 2, so that we were able to investigate the most useful part of the *ka–kr* diagram. A new type of lateral reproducer[7] satisfying the conditions assumed in the distortion analysis was employed.

The test grooves were cut on lacquer coated records. The cutting head used relies to some extent on the record material for damping and so did not yield a constant amplitude at all groove speeds. This was taken into account in constructing the solid "calculated" curves of Fig. 9, the larger amplitudes being measured optically while the smaller were found by measuring the relative reproducer outputs at the fundamental frequency. Measurements were made at various record radii for two record speeds and for three different recording levels. The results are shown in Fig. 9 and seem to provide a wholly satisfactory verification of the mathematical analysis.

The residual distortion levels of 2, 5 and 10 percent, shown at the right-hand side of Fig. 9, may be attributed to the recording equipment, and principally to the recording head itself. It is, however, obvious that for the smaller values of tangential groove velocity tracing distortion is the predominant factor, and that its magnitude has the calculated value. It may also be concluded that not only were the reproducer specifications satisfactorily met, but that the assumption made in the analysis regarding the negligibility of momentary deformations of the groove sidewalls was justified.

[7] Hunt and Pierce, Electronics, **11**, 9 (1938).

Conclusions and Recommendations

A method of reducing the effective surface noise level has been suggested, and in some cases utilized, which consists in predistorting the recording frequency characteristic in such a way that high frequency components are recorded at an increased level. Complementary equalization in the reproducing system restores the original balance and at the same time suppresses a portion of the surface noise generated in reproduction. The distortion contour chart provides a method of evaluating the effect on harmonic distortion of this type of alteration of the frequency response of the recording system. We have indicated above that in vertical cut records the total distortion is a linear function of the recorded amplitude so that it would appear at first sight that it would be permissible to enhance the high frequencies in recording, allowing the distortion to increase linearly, and in the complementary equalization not only restore the original tonal balance but also return the relative harmonic distortion to the original value it would have had without modification. This is indeed a useful method of reducing effective surface noise so long as its use does not increase the distortion beyond the range for which the second harmonic, which is the component varying linearly with amplitude, is the principal distortion factor. If significant distortion terms higher than the second occur, not only will they increase more rapidly, and hence not be proportionately removed by the reproducing equalization, but the higher terms will also contribute cross modulation products which would not have been present at the original distortion level. Inasmuch as the distortion level is already high for vertical reproduction under typical conditions of groove amplitude and stylus radius, it appears that the gain to be derived from a predistorting technique is rather limited. On the

other hand, in the case of lateral groove modulation, the total distortion increases with the square of the recorded amplitude so that an increase in distortion introduced by a modification of the recording frequency characteristic would not be compensated by complementary equalization in the reproducing system. It appears, therefore, that in this case the predistorting technique can never be employed unless the needle tip radius can be reduced to such an extent that tracing distortion is a negligible factor in the over-all distortion of the system.

The effect of cross modulation mentioned above should be emphasized in connection with the curves of Figs. 6 and 8. It is probable that we are seldom if ever interested in the harmonic distortion components accompanying fundamental frequencies higher than 4000 to 6000 cycles. On the other hand, the cross modulation products which accompany such distortion are of considerable interest and, although difficult to analyze accurately, are approximately of the same magnitude as the harmonic components themselves. Thus all of the distortion data presented above for frequencies higher than about 5000 cycles are to be interpreted as indicative of the magnitude of the cross modulation components. These components are observed aurally in wide range systems as a "burr" accompanying loud passages. The sum and difference tones, being inharmonic, are more objectionable than comparable harmonic overtones. Thus, even though the overtones of these high frequencies may be outside the transmission band of the reproducing channel, the distortion limits which should be imposed upon a high-fidelity system are more severe than for a system whose response is restricted to the lower frequencies. These considerations indicate that the 10 percent distortion limit assumed for the curves of Fig. 8 is too high to be acceptable if the over-all system response is to extend significantly above 5000 cycles.

A second conclusion can be drawn from the foregoing exhibition of the distortion performance of typical vertical and lateral recording conditions, based upon the broken-straight-line envelope of the peak power correction curve shown in Fig. 7. It may be seen from this curve that the low frequency peak amplitude falls off by slightly more than 6 db per octave for frequencies below 250. On the other hand, this is exactly the reduction in velocity amplitude which one seeks to gain by altering the standard cut from constant velocity to constant amplitude for frequencies below 250–300 cycles. The conclusion is that there is no real necessity for altering the character of the cut from constant velocity to constant amplitude at this low frequency. If the "standard" cut is maintained at constant velocity amplitude for the entire spectrum the greatest danger of "overcutting" would still occur in the neighborhood of 300 cycles, just as under the present "standard" conditions, but it would then become unnecessary to provide electrical equalization for the range below 300 cycles, as is at present required with high-fidelity recording and reproducing equipment.

A third conclusion and recommendation may be based upon a possible modification of the "standard" groove cross-sectional shape used in lateral recording. It seems almost certain that a lateral reproducer having the necessary vertical mobility assumed in the foregoing analysis can be designed to exert extremely light forces upon the groove wall. If this feature of the design can be carried far enough there is no reason why the needle tip radius may not be reduced to less than 1 mil. If these conditions are satisfied there is then no necessity for, (a) a total groove depth of 2.5 mils, or, (b) such a large rounded bottom portion of the groove. For example, a sharp bottomed (or very slightly rounded) groove 2 mils wide should be adequate to provide tracking for a reproducer capable of operating satisfactorily with a stylus tip radius of 0.75 mil. If the desired maximum amplitude of groove modulation be retained at 2 mils, there would be a net saving of some 40 percent of the available record surface, so that a groove pitch of 175 per inch could be used. Such groove spacing would allow as much as 30 minutes of recording on each side of a 16-inch, 33 r.p.m. transcription record, with no sacrifice in the present available recorded levels, and with a material reduction in total harmonic and cross modulation distortion compared with present transcription records. For a 12-inch, 78 r.p.m. record suitable for home use a total of 11 minutes of recording would be avail-

able with a similar reduction in total distortion as compared with current practice. These reductions in harmonic distortion and the gain in length of playing time stem principally from the reduction in radius of the needle tip and the consequent desirability of reducing the width of the recorded groove at the record surface. Table I exhibits a comparison of the playing times available with a 10 percent and a 5 percent distortion limit, for lateral cut records. The gain in playing time and usefulness of the convenient 12″—33 r.p.m. record is worthy of note, as is the fact that the lower the permissible distortion the greater is the advantage of the proposed narrow groove and small stylus tip.

To achieve comparable reductions in distortion for the present type of standard, round bottomed, lateral cut groove, Mr. Olney has suggested to us the possibility of using a needle tip having either an elliptical cross section presenting a small radius of curvature to the groove wall, or, alternatively, a needle tip section consisting of a flat circular disc perpendicular to the groove axis with the edges rounded to a small radius.

Because this study has enabled us to predict the conditions necessary to its success, we wish to call attention here to the system of controlled volume expansion illustrated in Fig. 10. This method has been proposed before as a means for avoiding volume distortion, but it does not appear to have made its way into the art as yet.

As the figure indicates, the only modification necessary in the recording technique is the introduction of a constant tone at a point in the system preceding the gain control which is used to compress the program material. This pilot tone must be of such a frequency that it is within the pass-band of the recorder and reproducer, but outside of the desired program band. In the diagram we suggest a 12 kc pilot tone, to be used with a 10 kc program channel. No other change need be made in the recording technique.

When such a record is reproduced with conventional equipment not responsive to the high frequency pilot tone the performance is entirely normal and user need not be aware that the record is in any way unusual. This seems to be an important feature since the pilot tone could be introduced in commercial records without impairing their value for use with existing phonographs. On the other hand, if the user wishes to take advantage of the enhanced volume range it is only necessary to employ a reproducer capable of responding to frequencies as high as that of the pilot tone, to segregate the pilot tone with a filter, rectify it, and apply it to the automatic volume control circuit of an amplifier similar to those ordinarily used in radio receivers. This automatic volume control operates to maintain a constant level of the pilot tone at the amplifier output. Since this condition is that which obtained during recording, the original volume range will have been restored. That this is possible may be made clearer by consideration of the fact that we now have two independent recorded and reproduced channels, operating in synchronism. By proper use of these two channels we are able to add their volume ranges while listening to the program material carried by one of them.

TABLE I.

Size and Speed of Record	Playing Time in Minutes			
	r=2 mils 100 grooves/inch	r=0.75 mil 175 grooves/inch	r=2 mils 100 grooves/inch	r=0.75 mil 175 grooves/inch
	10 percent distortion		5 percent distortion	
10″—78 r.p.m.	4.2	8.8	3.8	8.3
12″—78 r.p.m.	5.5	11.0	5.1	10.5
16″—78 r.p.m.	8.1	15.5	7.7	15.0
12″—33 r.p.m.	6.9	19.2	4.8	16.8
16″—33 r.p.m.	12.9	29.7	10.8	27.3

The *ka–kr* chart of Fig. 4 indicates that the high frequency pilot tone will be subject to considerable distortion (and cross modulation with the program material) unless it is recorded at a level substantially less than that of the program. In spite of this restriction the effective signal-to-noise ratio for the pilot channel may be as high as for the program channel if the control tone is separated out with a filter whose pass-band is no wider than necessary to guard against variations in turntable speed, and if the time constant of the automatic volume control circuit is large enough to smooth out the irregularities of surface noise.

In brief, such a system of controlled volume expansion can furnish an accurate complement

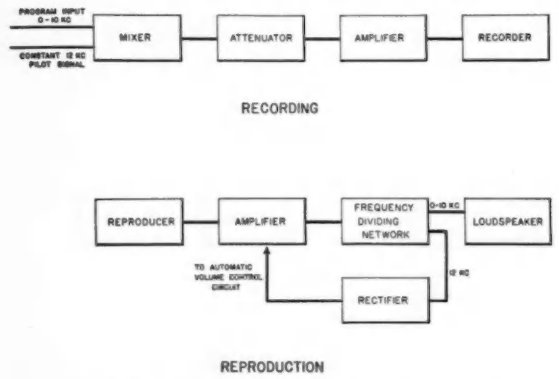

FIG. 10. Block diagram illustrating a system for controlled volume expansion, which is capable of restoring the full dynamic range of the original program material.

to the compression necessary in recording, and provide for the recreation of the desired program in its full dynamic range. At the same time, it should be emphasized that such records would remain as satisfactory as any of those in common use when reproducing facilities for expansion are not available.

As an alternative method of utilizing a second recorded channel, it may be pointed out that if it is undesirable to recreate the full dynamic range of the original material the control channel may be used to provide volume inflection while the program is recorded well above noise level at all times. While such records could only be used with reproducing equipment designed especially for them, they would provide phonographic reproductions entirely free from the audible effects of surface noise.

In conclusion it may be said that a principal result of this study has been the recognition and analysis of a large latent advantage, with regard to distortion, inherent in the lateral type of groove modulation. While not all of these indicated gains are realized by the present conventional lateral cut technique, we hope that new reproducer designs and a study of these geometrical relations will furnish some guidance for significant improvements in the fidelity and usefulness of disk records.

An Operational Treatment of Nonlinear Dynamical Systems

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(Received January 5, 1938)

When a mechanical system is undergoing vibrations whose amplitudes are small but not infinitely small, then the Lagrangian equations of motion are of a nonlinear character and the phenomenon is most complex. The Laplacian transform, or operational treatment of linear dynamical systems is being more and more widely employed in the treatment of linear systems leading to a set of ordinary differential equations or to partial differential equations of the linear type. The use of the operational method in nonlinear cases here presented appears to be new. For purposes of illustration, a system of acoustical interest will be considered although the method is general and yields interesting results in a wide variety of cases.

I. THE LAPLACIAN TRANSFORMATION

BEFORE proceeding, it may be well to review briefly the theory on which the modern operational calculus is based. Modern rigorous operational calculus is based upon the Fourier-Mellin integral equations. This set of equations state that if

$$g(p)=p\int_0^\infty e^{-pt}h(t)dt \tag{1}$$

then

$$h(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{g(p)e^{pt}dp}{p} \quad \text{where} \quad i=(-1)^{\frac{1}{2}} \tag{2}$$

and c is a real constant of such magnitude that the poles of the integrand lie entirely to the left of the path of integration. Following Van der Pol,[6] we shall for brevity write

$$g(p)\doteqdot h(t) \tag{3}$$

to express the above relationship between the functions $g(p)$ and $h(t)$. And we shall say that $g(p)$ is the operational equivalent of the function $h(t)$.

Many interesting and important theorems have been established[6] which make these integral equations most powerful tools in the solution of linear systems. It will now be demonstrated how the procedure may be modified so that nonlinear systems may be analyzed.

II. THE THEORY OF COMBINATIONS OF TONES OF MUSIC

To consider a concrete problem with which to illustrate the method, let us consider the combination of tones of music theory. Helmholtz in his *Sensations of Tone* stresses that in the slightly funnel shaped tympanic membrane and its connections, we have an unsymmetrical dynamical system and that the displacement of the system may be expressed by the equation

$$\ddot{x}+n^2x+hx^2=X(t) \tag{4}$$

where $X(t)$ represents the disturbing force. We shall investigate the joint action of two simple-harmonic forces and consider the equation in the form

$$\ddot{x}+n^2x+hx^2=a\cos(st)+b\cos(qt). \tag{5}$$

For completeness, let us assume the initial boundary conditions,

$$x=c, \quad \dot{x}=0, \quad \text{at} \quad t=0. \tag{6}$$

$$\text{Now let} \quad y(p)=p\int_0^\infty e^{-pt}x(t)dt \tag{7}$$

$$\text{or} \quad y(p)\doteqdot x(t), \tag{8}$$

that is, we set $y(p)$ in operational equivalence with $x(t)$.

If we multiply the equation (5) by $pe^{-pt}dt$ and integrate from 0 to infinity, we obtain, on integration by parts and using the boundary conditions (6).

$$(p^2+n^2)y=p^2c+ap^2/(p^2+s^2)$$
$$+bp^2/(p^2+q^2)-ph\int_0^\infty e^{-pt}x^2dt, \tag{9}$$

where we have made use of the well-known operational equivalent:

$$\cos(wt)\doteqdot p^2/(p^2+w^2). \tag{10}$$

Now let $$x=\sum_{n=0}^{\infty}x_n \tag{11}$$

and $$y=\sum_{n=0}^{\infty}y_n \tag{12}$$

with the condition that

$$y_i \doteqdot x_i, \tag{13}$$

that is, each y_i quantity is placed in operational equivalence with the corresponding x_i quantity. By multiplication:

$$x^2=x_0^2+2x_0x_1+x_1^2+2x_1x_2+x_2^2+\cdots.$$

Substituting into (9), we obtain

$$y=1/(p^2+n^2)\Big\{p^2c+ap^2/(p^2+s^2)+bp^2/(p^2+q^2)-ph\int_0^{\infty}e^{-pt}(x_0^2+2x_0x_1+x_1^2+2x_1x_2+)dt\Big\}. \tag{14}$$

Now let

$$y_0=1/(p^2+n^2)[p^2c+ap^2/(p^2+s^2)+bp^2/(p^2+q^2)]\doteqdot x_0 \tag{15}$$

$$y_1=-ph/(p^2+n^2)\int_0^{\infty}e^{-pt}x_0^2dt\doteqdot x_1, \tag{16}$$

$$y_2=-ph/(p^2+n^2)\int_0^{\infty}e^{-pt}2x_0x_1dt\doteqdot x_2 \text{ etc.} \tag{17}$$

On the supposition that h is less than n^2, the series may be proved to be most rapidly convergent. The convergence of series of this type has been investigated under fairly wide conditions.[3, 4]

The advantage of the operational process is that if one is equipped with a table of operational equivalents, one may write successive terms almost at a glance. To proceed, we have the operational equivalents,

$$ap^2/(p^2+n^2)(p^2+s^2)\doteqdot a[\cos(st)-\cos(nt)]/(n^2-s^2), \tag{18}$$

$$bp^2/(p^2+n^2)(p^2+s^2)\doteqdot b[\cos(qt)-\cos(nt)]/(n^2-q^2), \tag{19}$$

where we assume $n\neq s$ or $n\neq q$ so that the impressed forces are not in resonance with the system. Hence since

$$x_0\doteqdot[p^2c+ap^2/(p^2+s^2)+bp^2/(p^2+q^2)]/(p^2+n^2), \tag{20}$$

$$\therefore x_0=C\cos(nt)+A\cos(st)+B\cos(qt), \tag{21}$$

where $$A=a/(n^2-s^2),\quad B=b/(n^2-q^2),$$
$$C=c+a/(s^2-n^2)+b/(q^2-n^2). \tag{22}$$

Now,

$$x_0^2=C^2\cos^2(nt)+A^2\cos^2(st)+B^2\cos^2(qt)+2AC\cos(nt)\cos(st)+2BC\cos(nt)\cos(qt)+2AB\cos(st)\cos(qt). \tag{23}$$

We must now compute

$$y_1=-ph/(p^2+n^2)\int_0^{\infty}e^{-pt}x_0^2dt\doteqdot x_1. \tag{24}$$

We now make use of some well-known operational equivalents. For brevity, let

$$T(a)=1/(p^2+a^2), \tag{25}$$

then

$$\cos(at)\cos(bt)\doteqdot p^2/2[T(a+b)+T(a-b)], \tag{26}$$

$$\cos(at)\doteqdot p^2T(a), \tag{27}$$

$$\cos^2(at)\doteqdot\tfrac{1}{2}[p^2T(2a)+1] \tag{28}$$

and

$$T(a)T(b)p^2\doteqdot[\cos(at)-\cos(bt)]/(b^2-a^2). \tag{29}$$

Making use of these operational equivalents, we obtain

$$y_1=-hT(n)\{c^2/2[p^2T(2n)+1]+A^2/2[p^2T(2s)+1]+B^2/2[p^2T(2q)+1]+p^2AC[T(n+s)+T(n-s)]+p^2BC[T(n+q)+T(n-q)]+p^2AB[T(s+q)+T(s-q)]\}, \tag{30}$$

and interpreting the various operational equivalents, we obtain

$$\begin{aligned}x_1=&\cos(nt)\{hc^2/3n^2+hA^2/2n^2\\&-hA^2/(8s^2-2n^2)-hB^2/(8q^2-2n^2)\\&+hB^2/2n^2-hAC/(s^2+2sn)-hAC/(s^2-2sn)\\&-hBC/(q^2+2qn)-hBC/(q^2-2qn)\\&-hAB/[(s+q)^2-n^2]-hAB/[(s-q)^2-n^2]\}\\&+\cos 2nt(hc^2/6n^2)+\cos(2st)[ha^2/2(4s^2-n^2)]\\&+\cos(2qt)[hB^2/2(4q^2-n^2)]-h(A^2+B^2\\&+C^2)/2n^2+\cos(n+s)t[hAC/(s^2+2sn)]\\&+\cos(n-s)t[hAC/(s^2-2sn)]\\&+\cos(n+q)t[hBC/(q^2+2qn)]\\&+\cos(n-q)t[hBC/(q^2-2qn)]\\&+\cos(s+q)t[hAB/(s+q)^2-n^2]\\&+\cos(s-q)t[hAB/(s-q)^2-n^2].\end{aligned} \tag{31}$$

Higher order terms may be similarly obtained by using the next term in the series

$$y_2 \doteqdot -ph/(p^2+n^2)\int_0^\infty e^{-pt}2x_0x_1dt \doteqdot x_2. \quad (32)$$

Such terms contain h^2 as a coefficient and may be neglected on the supposition that h is less than n^2.

In this case, that of the tympanic membrane, the free period $2\pi/n$ is relatively long and the most important combination tone is the difference tone because of the greater magnitude of its coefficient.

III. Free Asymmetric Vibrations

Let us suppose a particle to be acted upon by an attractive force $F(r)$ depending upon the distance r from some point. If the particle be displaced a distance x, then the force experienced by the particle is

$$F(r+x)=F(r)+xdF/dr + (x^2/2)(d^2F/dr^2)+\cdots+. \quad (33)$$

Let us measure x from the position of equilibrium so that $F(r)=0$. Now if the displacement is such that the square term must be included, then the equation of motion of the particle is

$$m\ddot{x}+fx+gx^2=0 \quad (34)$$

where the coefficients f and g depend upon the nature of the force.

Or
$$\ddot{x}+n^2x+hx^2=0, \quad (35)$$

where
$$x^2=f/m, \quad h=g/m. \quad (36)$$

This is the equation (4) with a and b put equal to zero. If the boundary values are

$$\dot{x}=0 \quad \text{and} \quad x=c \quad \text{at} \quad t=0, \quad (37)$$

we have the solution given by (31) on placing $a=0$ and $b=0$ in the various quantities.

This equation, which is a special case of (4) has been treated by Hantaro Nagaoka.[5] His treatment makes use of elliptic functions. The equation (34) is of particular importance in the theory of seismic waves in that the presence of the nonlinear term gives rise to harmonics of the natural frequency.

IV. General Discussion of the Method

The method here presented is adequate for the solutions of equations of the type

$$\ddot{x}+k\dot{x}+hx+ax^2+2bx\dot{x}+c\dot{x}^2=f(t), \quad (38)$$

where a, b, c, h, and k are constants.

The free and forced vibrations may be easily obtained to any required degree of approximation. The advantage gained in the use of the operational method is the ease with which higher order terms may be written down as well as the manner in which boundary conditions may be advantageously expressed.

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Acoustic Spectra of Organ Pipes

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INTRODUCTION

ALTHOUGH the literature of acoustics is relatively rich in the field of vibrating strings, both with respect to the factors determining the frequency of vibration and, to a less degree, the harmonic structure of the tone, the corresponding problems in the case of organ pipes have not been adequately treated. This incompleteness is especially apparent in the field of partial structure of pipe tones. In fact, this condition would be expected to obtain because of the lateness of development of accurate experimental methods. Most organ builders have not had facilities or technique to make the analyses; while acoustical investigators with the proper technique have been engaged in more pressing problems. This paper is a report on the work done to date on the organ pipe problem in the physics laboratory of the University of Texas, and an outline of the problems on which work is now proceeding. The work has been done by the author and two graduate students, Mr. F. O. Bohls and Mr. Massengale White.

The history of organ pipe development may be found in standard books on the subject.[1] An excellent short description of the more common pipes is that of Leet.[2] The theory of the production of definite fundamental frequencies in flue pipes and in reed pipes and experimental tests of the dependence of fundamental frequency on various factors have been discussed by a large number of investigators. Benton[3] has derived certain relations for the frequencies of the partials of the chimney flute. Blaikley[4] showed experimentally that, in one case, the second partial was not harmonic to the resonant fundamental. However, experimental analysis of the partial structure of the several classes of organ tone, under proper acoustical surroundings and with properly calibrated equipment, has not heretofore been made.

THE METHOD

The organ pipe to be studied is placed on its wind chest at the top of a 24-foot tower outdoors. Sound from the pipe is incident on a calibrated ribbon microphone placed on a movable tower made of pipe and of variable height. The output of the microphone is amplified by a low-distortion amplifier, the output of which is analyzed by means of a General Radio type 636-A wave analyzer. This method is essentially the technique discussed by Wente[5] and Hall[6] (insofar as the final electrical analysis is concerned) and used by Saunders[7] in his work on violin tones, except that the apparatus is not automatic recording. Since this particular study is of the pipe in the steady state, and since the steady state is easily maintained in organ pipes, the lack of automatic recording is not a serious detriment.

Particular attention has been paid to the effect of possible reflections from the surroundings (tower, ground, etc.). With a pressure-actuated microphone, spurious reflections produced variations in amplitude, with motion of the analyzing microphone, of as much as 10 db. However, with the null plane of the ribbon microphone properly oriented, changing the distance from pipe to microphone from 4 feet to 12 feet produced relative changes in the analyses of less than 3 db. All comparative tests were made with the microphone located 9 feet from the organ pipe under test, on a line passing perpendicularly through the center of the pipe. In general, readings on the same pipe on several days checked to within 3 db, provided wind pressure and tuning remained constant.

PIPES ANALYZED

The analyses in this particular study have been confined to diapason, string, and chorus-

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[7] F. A. Saunders, J. Acous. Soc. Am. **9**, 2, 81 (1937).

reed pipes. Through the courtesy of four organ builders chosen at random, certain species of pipes of the above-named tonal varieties were made available for test. The author is particularly indebted to the Wicks Organ Company of Highland, Illinois; the Reuter Organ Company of Lawrence, Kansas; the W. W. Kimball Company of Chicago, Illinois; and M. P. Möller, Inc. of Hagerstown, Maryland for the loan of these pipes. In order to avoid specific comparison between these builders, even though definite criteria of excellence are lacking, the builders of these pipes will be referred to as *A*, *B*, *C*, and *D*, the order of lettering having no relation to the order of listing given above.

This particular division of the work has been confined to a study of the acoustic spectra of tones from the above-named tonal families, each named by its builder and each properly tuned to its normal frequency of 261.6 vibrations

TABLE I. *Pipe dimensions.*

Pipe	Diameter	Mouth Width	Mouth Height	Tuning	Nicks per Inch	Bearded	Pressure	Weight
			Geigen Diapason					
A	1.6″	1.0″	0.34″	Slit	15	No	4″	2.0 lb.
B	1.6	1.3	.38	Collar	16	No	4″	2.4
C	1.63	1.25	.38	Slit	20	No	5″	1.6
D	1.7	1.3	.44	Collar	19	Yes	6″	1.6
			Open Diapason					
	2.0″	1.5″	0.50″	Collar	14	No	4″	3.4 lb.
			Viole D'Orchestre					
A	0.88″	0.60″	0.22″	Slit	50	Yes	4″	0.40 lb.
B	.75	.53	.22	Slit	50	Yes	4″	.45
C	.90	.75	.24	Slit	27	Yes	5″	.65
D	.94	.63	.22	Slit	25	Yes	6″	.58
			Salicional					
A	1.00″	0.63″	0.20″	Slit	50	Yes	4″	0.40 lb.
B	1.25	.80	.22	Slit	35	Yes	4″	.80
C	1.20	.80	.25	Collar	23	Yes	5″	.75
D	1.12	.80	.25	Collar	20	Yes	6″	.75
			Reed Pipes					
Pipe	Bell Diameter	Boot Length	Shallot Slot Width	Shallot Slot Length	Shallot Slot Shape	Shallot End	Pressure	Weight
			Trumpet					
A	2.2″	7.1″	0.02–.10″	1.6 ″	1	Straight	4″	1.00 lb.
B	2.1	7.6	.06–.20	.86	2	Filled	4″	1.00
C	2.4	4.0	.02–.13	1.25	3	Straight	5″	1.25
D	2.4	6.9	.04–.13	1.00	1	Filled	6″	1.25
			Cornopean					
A	2.7″	7.3 ″	.02–.13″	1.6 ″	1	Straight	4″	1.60 lb.
X	3.0	6.25	.30–.30	1.7	4	Beveled	3.5″	1.35
C	2.5	4.0	.02–.13	1.13	1	Straight	5″	1.75
D	3.1	7.5	.05–.20	1.13	1	Filled	6″	2.30

Slot Shapes: 1—tapering, with straight sides, 2—"acorn shaped," 3—sides parallel, except for quick taper near rear end, 4—sides parallel. All pipes have sharp upper lip edges, except geigen *B* and the open diapason. All languids are relatively low, except geigens *A* and *B*, viole *A*, and salicionals *A* and *D*.

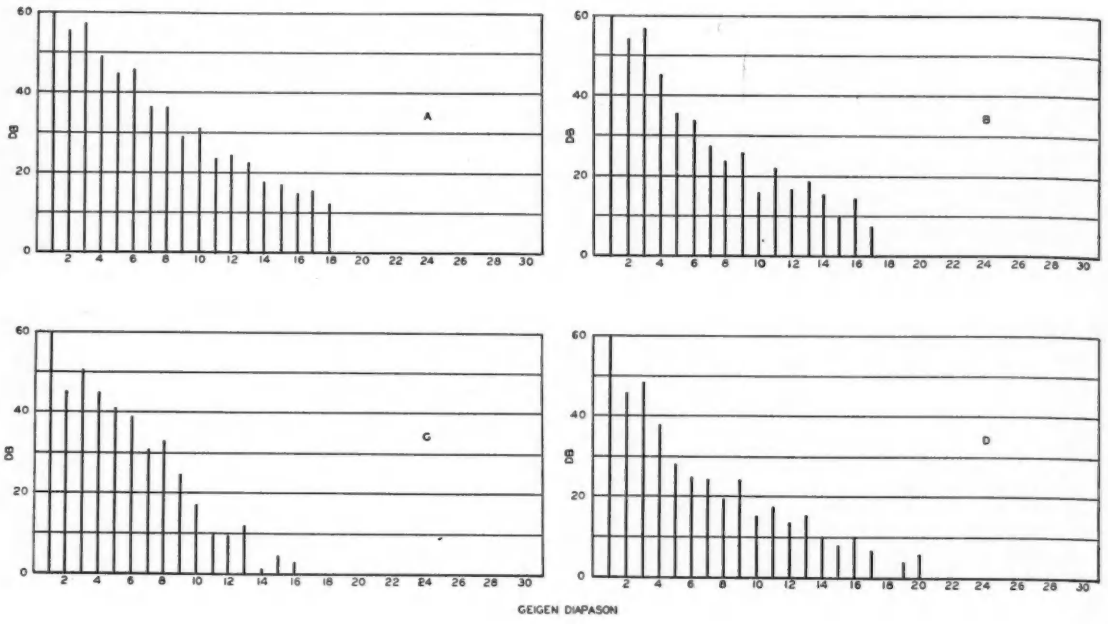

FIG. 1.

per second. There are, of course, variations in the mechanical structure of the several pipes in a given tonal family, and still larger variations between pipes in different tonal families. However, because of the large number of variables involved, even in a single species, the present report is restricted, for the most part, to analyses of the pipes as the builders furnished them, but tuned exactly and properly before testing. It may be remarked that only very minor adjustments were necessary on the pipes, even after shipment over considerable distance.

Study is also being made of the effects of the several variables in a given type of pipe, on both frequency and partial structure. These variables include such things as pipe length, cross section, shape, wind pressure, reed tongue and shallot structure, coupling between reed and pipe, angle of cone, pipe material, cut-up, mouth, nicking, and many others. Results of these investigations will be reported in due time. It is, of course, to be noted that only with the cooperation of the organ builder can such studies be readily made. It is, therefore, hoped that the continued cooperation of all the builders will permit a complete study of the many problems involved.

The various dimensions of the pipes tested are given in Table I.

EMISSION PATTERN

The emission patterns for both flue pipes and reed pipes were investigated by making analyses with the microphone occupying various positions in a vertical plane at a distance of 9 feet from the pipe. The patterns are of sufficient interest to warrant further study, particularly for points very near the pipe. In the plane in which all comparative measurements were made, flue pipes show maximum variations of approximately 8 db as the microphone is moved from the plane of the pipe toe to the plane of the pipe top. For smaller variations in position around the midpoint of this range, variations in the analysis are much smaller. For reed pipes, the corresponding maximum variation is approximately 3 db. The difference in patterns for flues and reeds is largely due to the emission from the flue mouth, which, of course, is not present in reed pipes. The location of the microphone was maintained essentially constant during all final analyses, at a perpendicular distance of 9 feet from the center of the pipe.

GEIGEN DIAPASON

In Fig. 1 are shown the spectra of four geigen diapason pipes. Pipes *A* and *B* are strikingly similar in the first four partials; above No. 4, *A* shows greater intensities in practically all

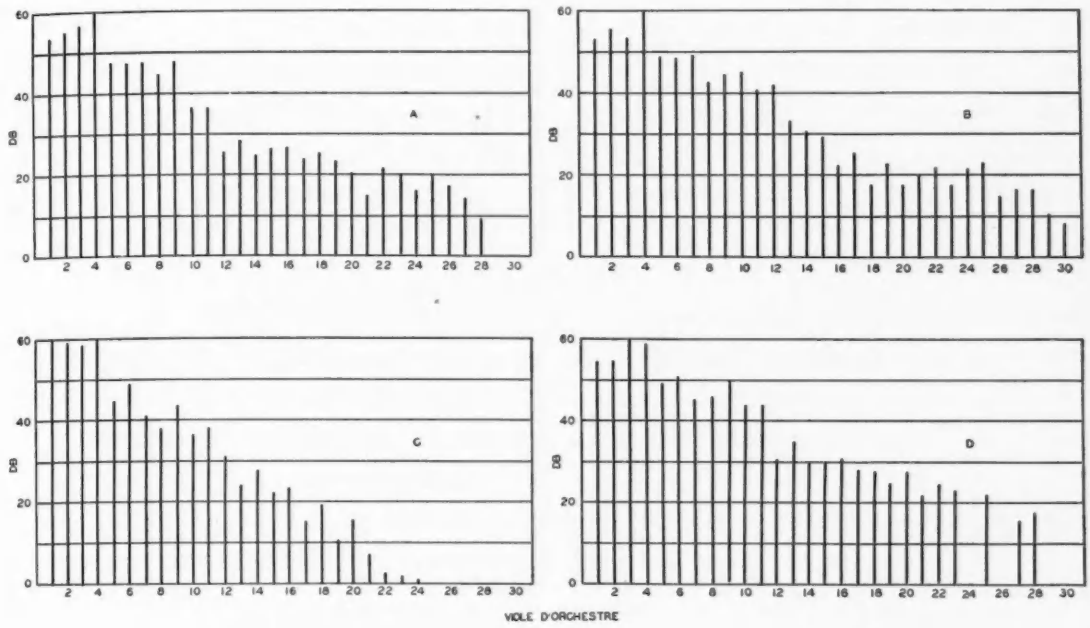

FIG. 2.

cases. It is to be noted, from Table I, that *A* has a narrower, lower mouth than *B*, is lighter, and has a slit tuner. Both *C* and *D* have smaller second and third partials than *A* and *B*. *C* drops off more rapidly than the others above the tenth; but *C* is physically very similar to *B* except in its finer nicking, higher pressure, slit tuner, and smaller weight. One could not conclude, therefore, that lighter weight, higher pressure, slit tuning, and the sharp upper lip that this pipe has, produce more harmonic development, so long as the mouth remains constant; rather, the reverse would seem to hold, unless the difference can be ascribed to a slight difference in nicking or to a more abrupt curvature of the lower lip of *B* (dubbing). *D* is a light pipe, with higher pressure, wide mouth, largest cut-up of all; *D* is weaker than *A* and *B* in partials 2, 3, 4, 5, 6, 7, and 8. It is apparent that the number of variables is so large that a much more detailed study of specially-built pipes is needed to break down the problem.

In Fig. 7 is plotted an "average analysis" of the four geigen diapasons. The first partial is predominant (as in each pipe); the second and third are less than 10 db lower than the first, with the third the stronger in all cases. Following a fourth at 15 db below the first, there occur two pairs of nearly equal amplitude (5–6 at 22 db down and 7–8 at 29 db down), followed by a declining sequence which decays almost linearly.

VIOLE D'ORCHESTRE

In Fig. 2 are given analyses of four viole d'orchestre pipes, with the average analysis in Fig. 7. The first eleven partials are, as a group, of high amplitude as compared with the geigen diapason. In all pipes the first four partials are outstanding. *C* drops off more rapidly than the others after No. 4, but is particularly strong in 1, 2, and 3. *C* has the widest and tallest mouth and coarse nicking; it is also heavy. On the other hand, *C* falls off more rapidly than the others in the highest partials and is somewhat like *C* of the geigen diapason, by the same builder. *A* and *B* are very similar, with *A* having the advantage in 8 and 9 and *B* having the advantage in 11, 12, 13, 14, and 15. *A* and *B* are constructionally similar, *B* being smaller and having a narrower mouth. The number and amplitude of upper partials is much greater than in the geigen diapason, and the tone may be described as "keen," faintly reminiscent of a violin. Experimentally, the tone is very considerably altered when partials above the eighth are removed (by electrical filter). All string-tone pipes, such as these violes, show random, al-

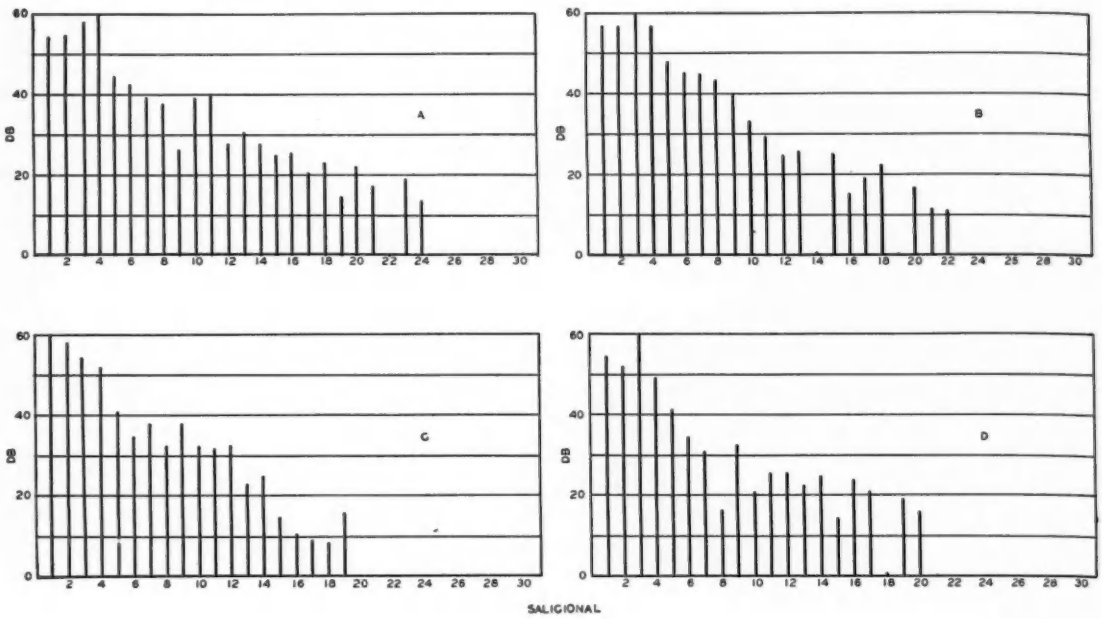

FIG. 3.

though small, frequency and intensity fluctuations while emitting sound in the "steady state." The tone is, consequently, "elusive" in nature, rather than forceful.

It is to be noted in *D* that partials 3, 6, and 9 stand out somewhat above the adjacent ones and that this tendency also appears in *A*, *B*, and *C*, although the fourth is predominant in the latter pipes.

SALICIONAL

In Fig. 3 are spectra of four pipes of the salicional type, sometimes described as a small diapason but more often thought of as a string-

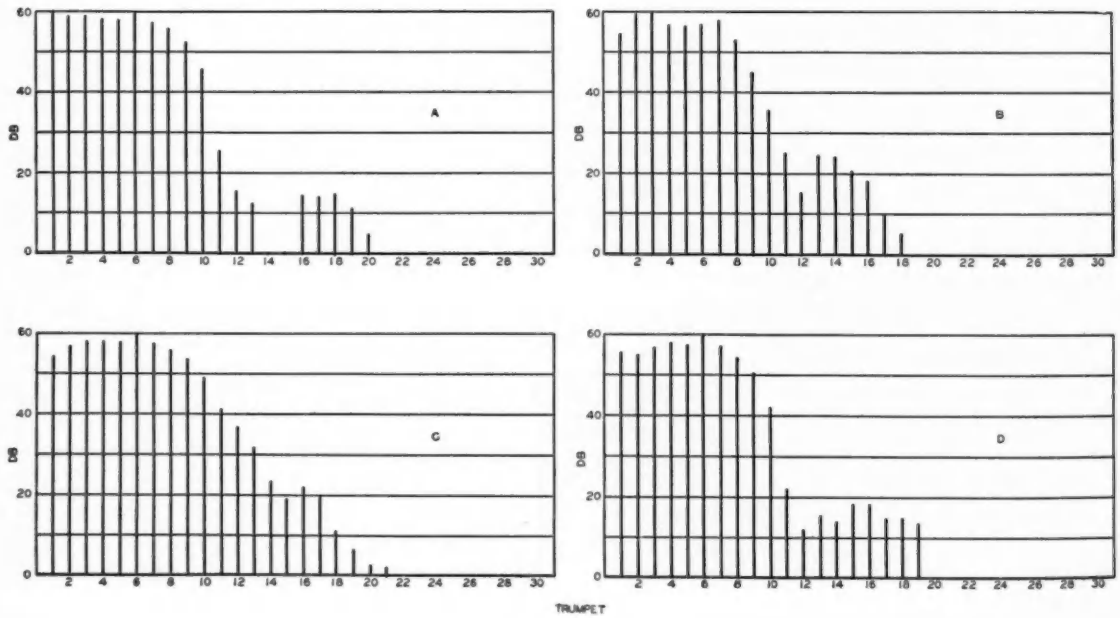

FIG. 4.

toned pipe. Pipe *A* is strikingly similar to viole d'orchestre *A* by the same builder, the differences being above the sixth partial. Dimensionally, salicional *A* differs from viole d'orchestre *A* principally in that salicional *A* has a greater diameter. One would conclude that the larger scales of string pipes give reduced amplitudes of the higher partials, other factors being constant. A similar condition exists for the *B* pipes, except that slight changes are also evident in the first four partials. It is to be noted that not only is salicional *B* of larger scale than viole d'orchestre *B*, but also that the salicional has a wider mouth, coarser nicking, and greater weight. Salicional *B* has fewer highest order partials than salicional *A*, although the companion violes were very nearly the same. The wide mouth, coarse nicking, and greater weight apparently combine to reduce high harmonic amplitudes in these pipes. *D* is similar to *B*, except for coarser nicking in *D*, higher pressure, and slightly smaller diameter. *D*'s mid-range partials and No's 2 and 4 are lower than *B*'s.

Salicional *C* is seen to be closer in analysis to the geigen diapason than are *A*, *B*, and *D*. These latter three, particularly *A* and *B*, are more nearly violes. *D* resembles the viole in the low numbered partials, the diapason in the mid-section, and the viole in the upper section. The average salicional in Fig. 8 resembles the viole more than the diapason, but is definitely on the diapason side.

Trumpet

In Fig. 4 are spectra of four conical trumpets. All are characterized by ten very strong initial partials, with *A*, *C*, and *D* having a predominant sixth. *B* and *C* are very similar, although their shallots are very different. *A* and *D* have straight-sided shallot slots; *D* has higher pressure and a slight filling of the shorter shallot end. Their spectra differ mainly in that the first five partials of *A* are stronger than in *D*. *B* was named "cornopean" by its builder, but its spectrum is distinctly that of a trumpet, if one is to define the term "trumpet" by the average of *A*, *C*, and *D*. *C* differs from *D* mainly in partials above the ninth, *C* being better developed in this upper region. These two pipes weigh the same and have the same bell; *C* is on lower pressure and has a longer and wider slot in the shallot, with little taper except near the end, and a shorter boot. The shallot is responsible for most of the difference between *C* and *D*.

Cornopean

In Fig. 5 are acoustic spectra of four cornopeans (reed pipes with conical horns). *B* has

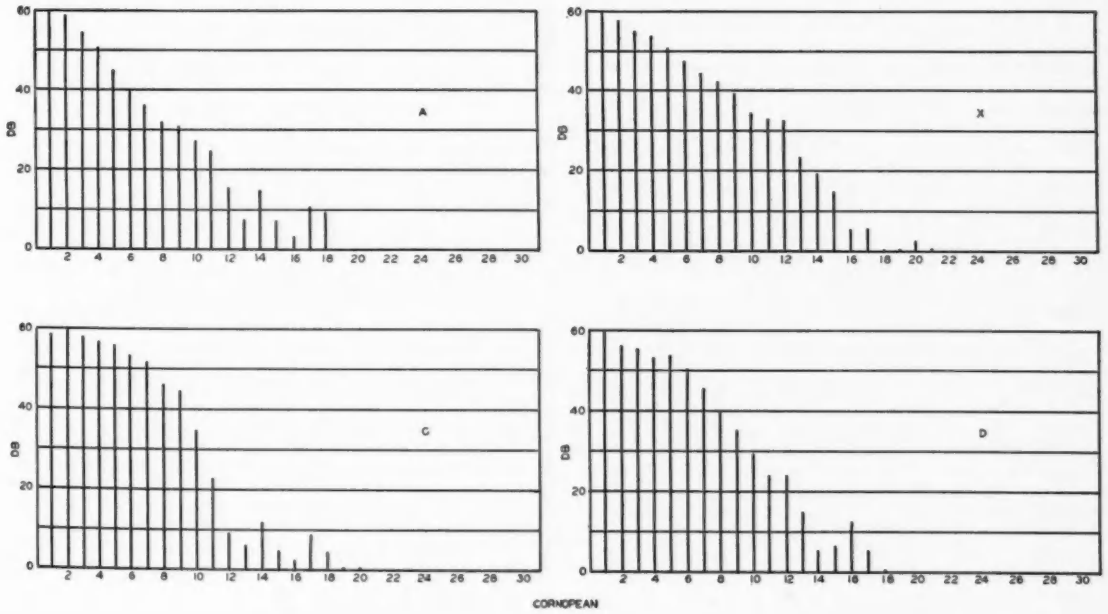

Fig. 5.

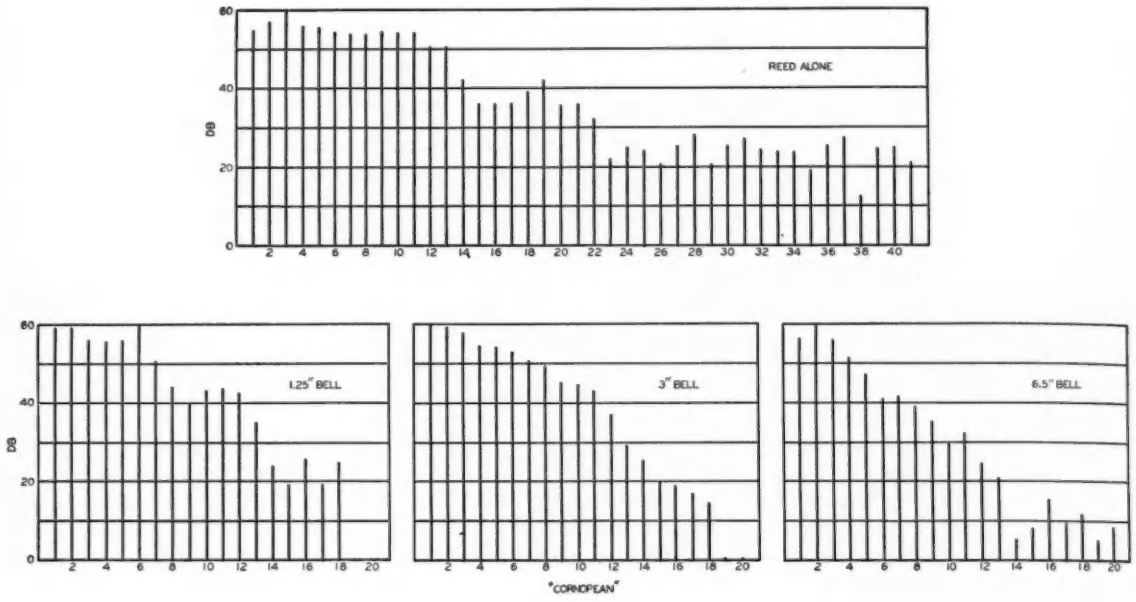

FIG. 6.

been replaced by *X*, a very old pipe of unknown make. The cornopeans differ from the trumpets, even though they are constructionally similar, in that cornopeans have a predominant fundamental, a second partial nearly as strong as the fundamental, followed by a series of partials decreasing in intensity uniformly with partial number. *A* gives a very pleasing, "rich-but-round" tone, whereas *X* is penetrating. The shallot slot of *X* is rectangular and large and the end of the shallot is beveled; the shallot of *A* is wedge-shaped and much narrower. Cornopean *A* is radically different in its spectrum from trumpet *A* by the same builder, the trumpet exhibiting better developed partials below the eleventh. The trumpet has a smaller bell and a slightly narrower slot in the shallot. It would appear that the bell diameter is a very important factor in determining partial structure. Cornopean *C* and trumpet *C* are more nearly alike than the two *D*'s; the *C*'s have nearly the same diameter of bell but differ in shallot construction and weight. Cornopean *D* is not strikingly different from trumpet *D*, when compared with the *B*'s, even though cornopean *D* has a much larger bell than trumpet *D*. But the larger bell is also accompanied by a somewhat larger shallot slot which probably prevents the expected rapid decay in the mid-range partials. Cornopeans *D* and *X* have nearly the same bell, but the shallot of *X* has a larger, straight slot and beveled tip, which factors seem to increase amplitudes of partials above the eighth.

In a preliminary investigation of the effect of bell diameter on partial structure, several conical horns were fitted, in turn, on the reed base of a pipe similar to pipe *X*. The results are shown in Fig. 6. For comparison purposes, only twenty partials are plotted, except for the reed without its horn (a kinura-like tone). In fact, the reed alone generates sixty or more partials above the zero level of the graphs. The effect of the large bell in reducing higher partials is apparent, and this result checks with the individual analyses of *A*, *X*, *C*, and *D* in Fig. 5. The 1.25″ bell horn changes partials above the sixth very little from those in the 3″ bell. The smallest horn does, however, emphasize partials 1, 2, and 6, with an accompanying depression of 3, 4, and 5. The tone ceases to be a chorus-reed tone and approaches that of an orchestral oboe. In fact, preliminary analyses of the organ orchestral oboes of three builders give results very similar to those in Fig. 6 with the small bell. Further experiments indicate that the higher frequency partials in the reed pipes are focused along the axis of the conical pipe, as one would expect.

The "end analysis" is thus very different from the "normal analysis." It is to be noted that Cloud[8] found little concentration of energy along the axis of the conical horn in an organ pipe.

Envelopes

In Fig. 9 are plotted "envelopes" of the acoustic spectral lines given in the preceding charts, Fig. 7 and Fig. 8. The exact meaning of these envelopes is doubtful with regard to their importance in hearing, and their interpretation must await judgment tests on the importance of individual partials to the hearing mechanism. Work on such tests is now in progress in this laboratory under the direction of Professor L. A. Jeffress of the department of psychology. In general, it appears that there is some importance to be attached to the slope of the envelope as well as to the location of peaks. For example, the open diapason and the geigen diapason of Fig. 9 have very similar envelopes, the former of greater slope than the latter. The open diapason is also a less brilliant and a smoother tone than the geigen. Similarly, the salicional has a greater slope than the viole d'orchestre over its descending portions; while the viole is the more incisive of the two. Further, the trumpet is much more brilliant than the cornopean and this characteristic seems to be reflected in the envelopes of the two.

[8] J. H. Cloud, Phys. Rev. **22**, 73 (1923).

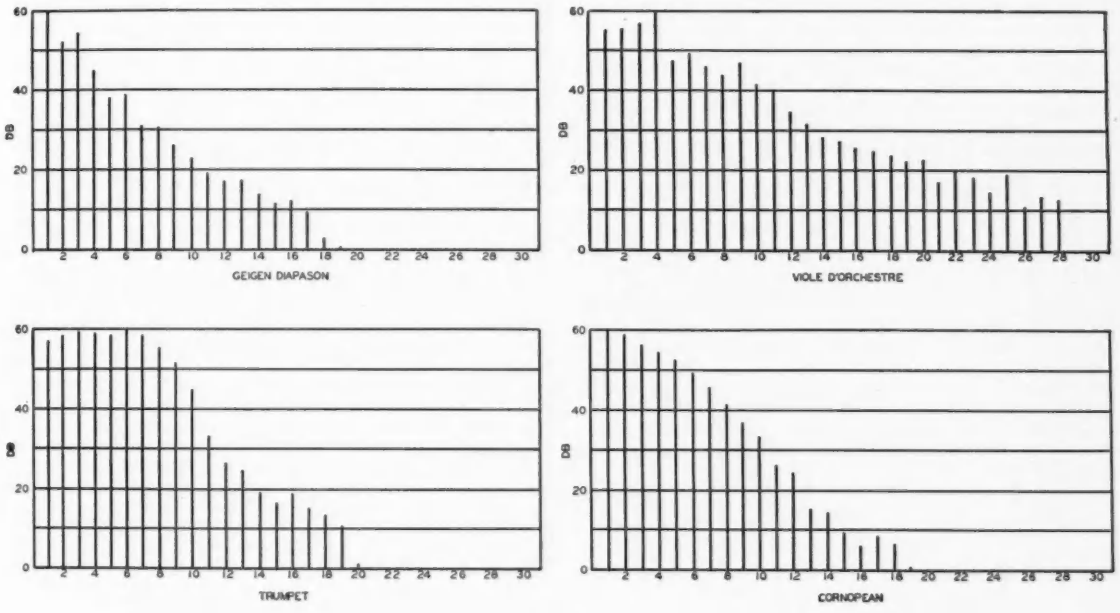

Fig. 7.

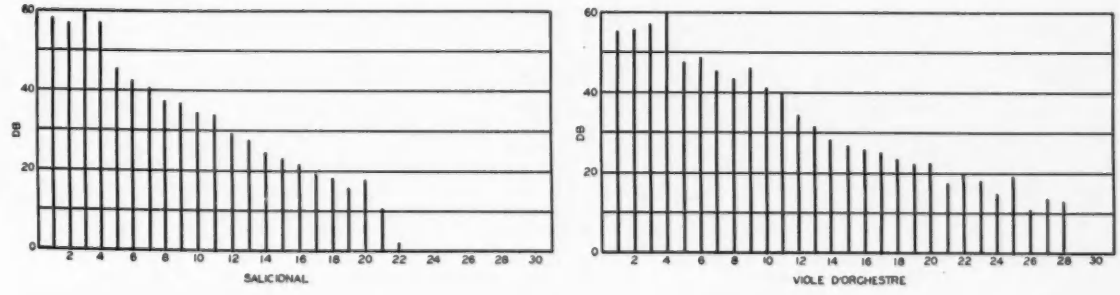

Fig. 8.

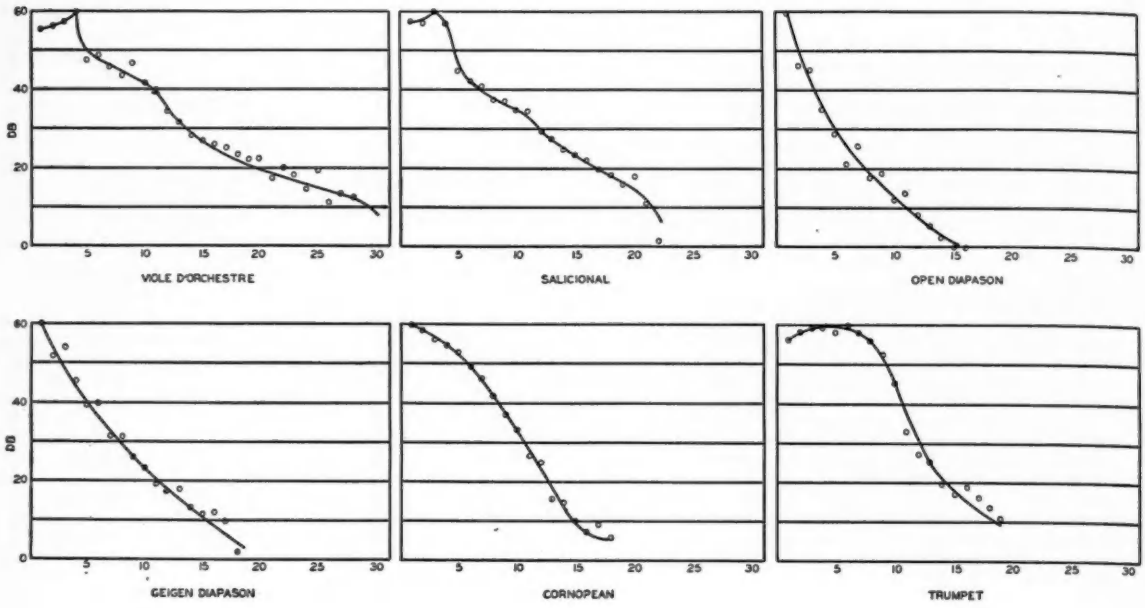

FIG. 9.

FREQUENCY OF VIBRATION

Measurements show that the fundamental frequency of each of the flue pipes analyzed (261.6 vibrations per second) is approximately 10 vibrations per second higher than the measured resonant frequency of the pipe.

The fundamental frequency of a reed pipe is a function of several variables. However, when both reed tongue and conical pipe are separately tuned to the same frequency and are then coupled together, the resulting frequency is a function of the bell diameter, provided other factors are constant. For one reed pipe tested, the frequency is given by the equation:

$$F = -53/d + f$$

when the tuning is as stated above. In this equation

F = frequency of the reed pipe,
d = diameter of conical bell, in inches,
f = frequency of conical pipe and reed tongue separately.

Further work on this problem is in progress. In general, over the customary range of tuning, it may be said that the frequency of a reed pipe is lower than that of either the reed tongue or conical pipe.

Measurements of the frequencies of the upper partials have been made for the pipes tested in this series, under normal conditions of blowing, but, at present, only with an accuracy of 0.5 percent. The partial frequencies are found to be harmonically related within this limit. Further work on this point is also in progress.

Finite Solid Acoustic Filters

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(Received March 28, 1938)

Calculation is made of the transmission of harmonic compressional waves through a finite solid filter structure consisting of a metal rod loaded at equal intervals with heavy metal collars. The theory is then tested experimentally for the special case where the finite structure (consisting of 6 sections) is terminated by a finite rod with a free end. The structure acts as a low pass filter with a cut-off at about 2000 cycles and no further transmission up to 12,650 cycles. The transmission in the transmission band shows a series of peaks whose positions, with one exception, are accurately verified experimentally.

INTRODUCTION

FINITE filters in air have been studied theoretically by Mason[1] and more recently by Lindsay.[2] The filtration of compressional waves in solids under a great variety of conditions has been investigated by Lindsay and his collaborators.[3] It is the aim of the present paper to study such filtration experimentally. In order to compare theoretical results with measur ments it is desirable to apply the theory to the ıctual finite filters used.

THEORY

We confine our attention to the case in which a solid rod loaded at equal intervals with heavy masses is inserted in an acoustic line with an input impedance Z_i and a terminal impedance Z_t. In the experimental arrangement the masses are heavy collars as indicated in Fig. 1. The

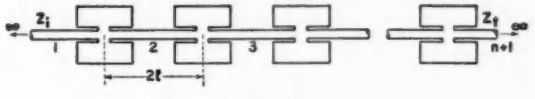

FIG. 1.

length of each section is $2l$ and the area of cross section is S. The mass of each load is m. We shall suppose there are n sections. Referring to the analysis of the infinite filter of this type,[4] we have for the volume current and excess tension at the end of the structure, labeled with the subscript $n+1$ (see Fig. 1), in terms of the corresponding quantities at the previous mid-section point

$$\begin{aligned} \dot{X}_{n+1} &= A\dot{X}_n + iC/Z \cdot T_n, \\ T_{n+1} &= AT_n + iBZ\dot{X}_n, \end{aligned} \tag{1}$$

where $Z=\rho_0 c/S$ (ρ_0 = mean density of rod material, c = velocity of compressional waves in the rod) and

$$\begin{aligned} A &= \cos 2kl - \omega m/2ZS^2 \cdot \sin 2kl, \\ B &= \sin 2kl + \omega m/ZS^2 \cdot \cos^2 kl, \\ C &= \sin 2kl - \omega m/ZS^2 \cdot \sin^2 kl. \end{aligned} \tag{2}$$

Inspection of (2) discloses that

$$A^2+BC=1.$$

Letting $A=\cos W$, $B=(B/C)^{\frac{1}{2}} \cdot \sin W$, $C=(C/B)^{\frac{1}{2}} \cdot \sin W$, we finally obtain by the use of mathematical induction

$$\begin{aligned} \dot{X}_{n+1} &= \dot{X}_1 \cos nW + i(C/B)^{\frac{1}{2}} \cdot T_1/Z \cdot \sin nW, \\ T_{n+1} &= T_1 \cos nW + i(B/C)^{\frac{1}{2}} \cdot \dot{X}_1 Z \cdot \sin nW. \end{aligned} \tag{3}$$

These relate terminal quantities to those at the first mid-section point (i.e. at 1 in Fig. 1).

Let us now suppose that to the right at 1 there are incident and reflected harmonic waves of frequency $\nu=\omega/2\pi$. If the incident and reflected volume currents are $\dot{X}_0$ and $\dot{X}_0'$, respectively, with the corresponding tensions T_0 and T_0', the boundary conditions become

$$\dot{X}_1=\dot{X}_0+\dot{X}_0', \quad T_1=Z_i(\dot{X}_0'-\dot{X}_0). \tag{4}$$

On substituting from (4) into (3), replacing T_{n+1} by $-Z_t\dot{X}_{n+1}$ and eliminating $\dot{X}_0'$, we finally get

[1] W. P. Mason, Bell Syst. Tech. J. **6**, 258 (1927).
[2] R. B. Lindsay, J. Acous. Soc. Am. **8**, 211 (1937).
[3] Cf. J. Acous. Soc. Am. **8**, 42 (1936).
[4] Lindsay and White, J. Acous. Soc. Am. **4**, 155 (1932).

$$\frac{\dot{X}_{n+1}}{\dot{X}_0}=\frac{2}{(1+Z_t/Z_i)\cos nW+i\sin nW/\sin W\cdot(Z/Z_i\cdot B+Z_t/Z\cdot C)}. \tag{5}$$

From this it follows that if we measure the power transmission P_r through the filter by the ratio of the total average rate of flow of energy *out of* the filter into the terminal impedance Z_t to the total average rate of flow *into* the filter, we have

$$P_r=Z_t|\dot{X}_{n+1}|^2/Z_i|\dot{X}_0|^2$$

$$=\frac{4}{(2+Z_i/Z_t+Z_t/Z_i)\cos^2 nW+Z_i/Z_t\cdot\sin^2 nW/\sin^2 W\cdot(Z/Z_i\cdot B+Z_t/Z\cdot C)^2}. \tag{6}$$

It is here assumed that the impedances Z_i and Z_t are real. The simplest case is that in which $Z=Z_i=Z_t$. Physically this corresponds to the insertion of the filter in an infinite solid rod of the same material as the filter rod itself and of the same area of cross section. In this case P_r becomes

$$P_r=\frac{1}{\cos^2 nW+\sin^2 nW/4\sin^2 W\cdot(B+C)^2},$$

which can be immediately put into the simpler form

$$P_r=\frac{1}{1+\omega^2m^2/Z^2S^4\cdot\sin^2 nW/4\sin^2 W}. \tag{7}$$

It is clear from an inspection of (7) that when $|\cos W|\leq 1$, so that $|\sin nW|\leq 1$ and $|\sin W|\leq 1$, P_r can become very close to unity, i.e. the transmission may be large. On the other hand, for $|\cos W|>1$, the sines become hyperbolic functions and as n increases the denominator in P_r will increase. Thus we have a succession of transmission and attenuation regions following with certain important variations of detail the bands of the corresponding infinite filter.

The power transmission ratio defined and calculated above is the one usually understood in acoustical measurements. It is not the only possible one, however, nor is it the easiest to realize in actual measurement. Thus, it is probably simpler to place intensity measuring devices (see the Experimental Procedure for details) at 1 and $n+1$, i.e. *before* and *after* the filter, and make measurements simultaneously. If we take the ratio of the latter to the former we should not expect to get P_r. Rather the result may be written (recalling that $\dot{X}=S\dot{\xi}$)

$$P_r'=|\dot{\xi}_{n+1}|^2/|\dot{\xi}_1|^2, \tag{8}$$

it being understood that the measuring instrument records something proportional to the actual displacement velocity or its square. We can compute P_r' from Eq. (3) by replacing T_{n+1} by $-Z_t\dot{X}_{n+1}$ and eliminating T_1. This yields

$$P_r'=S_i^2/S_t^2\cdot\frac{1}{\cos^2 nW+Z_t^2/Z^2\cdot C^2\sin^2 nW/\sin^2 W}, \tag{9}$$

where S_i and S_t are the cross-sectional areas of the rods to the left and right of the filter respectively. For the special case considered above where $S_i=S_t$ and $Z_i=Z_t=Z$, (9) reduces to the form

$$P_r'=1\Big/\left(1-C\omega m/ZS^2\cdot\frac{\sin^2 nW}{\sin^2 W}\right). \tag{10}$$

A comparison of (10) with (7) indicates that the pass and attenuation bands will agree approximately but that there will be certain differences. In particular we see that while $P_r\leq 1$ always, P_r' can exceed unity. Thus P_r', though a convenient measure of the transmission, is not a genuine transmission ratio. For m/ZS^2 small and for low frequencies for which the difference between B and C is small, inspection indicates that P_r' is approximately equal to P_r. The ratio

$$P_r/P_r'=\frac{1-C\omega m/ZS^2\cdot\sin^2 nW/\sin^2 W}{1+\omega^2m^2/Z^2S^4\cdot\sin^2 nW/4\sin^2 W} \tag{11}$$

is valuable for the estimation of P_r when P_r' has been measured experimentally.

The change of phase introduced by the filter can be obtained by writing

$$\frac{\dot{X}_{n+1}}{\dot{X}_0}=\frac{|\dot{X}_{n+1}|}{|\dot{X}_0|}e^{i\theta}. \tag{12}$$

It then follows at once from (5) that the phase angle θ is given by

$$\tan\theta=-\frac{\tan nW\cdot(Z/Z_i\cdot B+Z_t/Z\cdot C)}{\sin W\cdot(1+Z_t/Z_i)}; \tag{13}$$

For the special case $Z=Z_i=Z_t$ and for B close in value to C, θ approaches $-nW$ as would be expected from the infinite filter theory.[2]

The insertion of the finite filter into an infinite line corresponding to the P_r and P_r' values given in (7) and (9) provides considerable experimental difficulty, for one has to try to make the actually finite line after the filter behave as if it were infinite, i.e. prevent as far as possible the existence of reflection. Experimentally a much more easily realizable situation is that in which the filter is terminated by a *finite* rod. Let the length of the terminating rod be l_t and its area of cross section be S_t. If the density of the material composing it is ρ_{0t} and the velocity of sound in it c_t, we can write for the impedance at the end of the rod[5] away from the filter

$$Z_{tl}=\frac{Z_t\cos k_t l_t-iZ'\sin k_t l_t}{\cos k_t l_t-iZ_t/Z'\cdot\sin k_t l_t},$$

where $Z'=\rho_{0t}c_t/S_t$ and Z_t=terminal impedance of the filter as used in the previous discussion. Solving for Z_t in terms of Z_{tl} yields

$$Z_t=-T_{n+1}/\dot{X}_{n+1}=\frac{Z_{tl}\cdot\cos k_t l_t+iZ'\cdot\sin k_t l_t}{\cos k_t l_t+iZ_{tl}/Z'\cdot\sin k_t l_t}. \tag{14}$$

If the far end of the rod is rigidly clamped, $Z_{tl}=\infty$ and (14) yields

$$Z_t{}^c=-iZ'\cot k_t l_t. \tag{15}$$

If on the other hand, the far end is free, $Z_{tl}=0$ and

$$Z_t{}'=iZ'\tan k_t l_t. \tag{16}$$

We now get the ratio $\dot{X}_{n+1}/\dot{X}_1$ from Eq. (3) as follows

$$\dot{X}_{n+1}/\dot{X}_1=\frac{1}{\cos nW+iZ_t/Z\cdot(C/B)^{\frac{1}{2}}\cdot\sin nW}. \tag{17}$$

Let us take the case where $Z_t=Z_t{}^c$. Then

$$\dot{X}_{n+1}/\dot{X}_1=\frac{1}{\cos nW+Z'\cot k_t l_t/Z\cdot(C/B)^{\frac{1}{2}}\cdot\sin nW} \tag{18}$$

and for the transmission P_r' we get

$$P_r'=S_i{}^2/S_t{}^2\cdot\frac{1}{(\cos nW+Z'\cot k_t l_t/Z\cdot(C/B)^{\frac{1}{2}}\cdot\sin nW)^2}. \tag{19}$$

In the case where $Z_t=Z_t{}'$, the transmission becomes

$$P_r'=S_i{}^2/S_t{}^2\cdot\frac{1}{(\cos nW-Z'\tan k_t l_t/Z\cdot(C/B)^{\frac{1}{2}}\cdot\sin nW)^2}. \tag{20}$$

When $|\cos nW|>1$, we must replace $\cos nW$ by $\cosh nW_i$ and $\sin nW$ by $i\sinh nW_i$ where W_i is the imaginary part of the complex angle W. It is clear, however, that in this case C and B are of opposite sign and consequently $(C/B)^{\frac{1}{2}}$ is purely imaginary. The form of (19) and (20) therefore remains precisely the same with $\cosh nW_i$ for $\cos nW$ and $\sinh nW_i$ for $\sin nW$. Of course, it will be noted that for certain frequencies P_r' will become infinite, reflecting our neglect of irreversible dissipation.

Experimental Procedure

The experimental work was undertaken to test the theoretical transmission through a solid filter like that shown in Fig. 1. The filter used consisted of a rod of $\frac{1}{4}$-inch cold drawn steel (density 7.83 grams/cm^3) loaded with steel cylinders each weighing 792 grams and separated from each other by 20.32 cm. Thus $n=6$ and $l=10.16$ cm. The velocity of compressional waves in the steel rod was taken to be $c=5140$ meters/sec. At first the attempt was made to test the behavior of the filter in an equivalent infinite line. This was simulated experimentally by

[5] See, for example, Stewart and Lindsay, *Acoustics*, p. 138, for the analogous case of air in a cylindrical tube.

allowing the rod beyond the filter to extend about 30 feet through a box of oiled sand. It was hoped in this way to reduce the reflected waves from the terminus of the line. This worked fairly well and the cut-off frequency of the filter was found to agree rather well with that predicted by the theory. However the actual transmission in the transmission band was observed to deviate considerably from the theoretically predicted transmission, particularly at low frequencies. It was therefore decided to abandon this method of measurement as a precise test of the theory and concentrate on the case of the finite terminus, i.e., try to verify Eqs. (19) and (20). As a matter of fact the rigidly clamped terminating rod is difficult to achieve experimentally. Consequently the case $Z_t = Z_t'$ was the one actually employed (with $Z' = Z$).

The experimental arrangement is shown diagrammatically in Fig. 2. The driving unit C is fed with the amplified output of a beat frequency oscillator (BFO with the accompanying amplifier A_1 and transformer T). This driving unit consists of a permanent magnet six inches long and $\frac{1}{4}$-inch in diameter one end of which projects into a coil carrying the amplified output of the oscillator. The other end of the magnet is fastened to the acoustical transmission line and the magnet as a whole is considered a part of the line.

Piezoelectric phonograph pick-ups P_1 and P_2 are placed at each end of the filter and measured directly, through a meter V at the output of the amplifier A_2, the relative displacement velocities $\dot{\xi}_1$ and $\dot{\xi}_{n+1}$ in turn by the use of a double throw switch.

The frequency exciting the system is determined by superposing a part of the output of the beat frequency oscillator on a standard 100-cycle wave (produced by the low frequency oscillator LFO) in the oscillograph O and observing the Lissajous figures produced. Measurements were taken at intervals of 50 cycles in the range from 50 to 12,000 cycles.

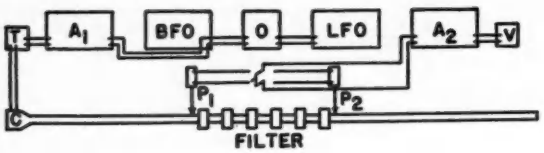

FIG. 2.

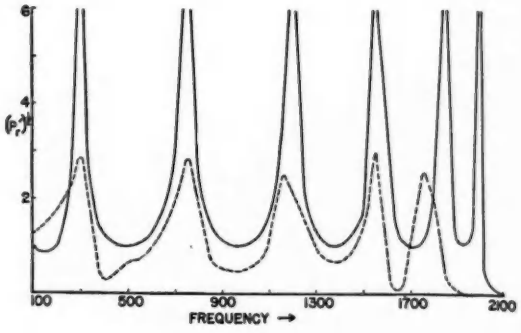

FIG. 3.

RESULTS AND DISCUSSION

The comparison between theory and experiment is shown in Fig. 3 in which the full line represents $(P_r')^{\frac{1}{2}}$ as computed from Eq. (20) while the dotted line corresponds to the measured ratio $|\dot{\xi}_{n+1}|/|\dot{\xi}_1|$. As one expects in a finite filter the transmission in the transmission band is by no means uniform but is marked by definite peaks. Save at the very end of the band the experimental peaks agree in frequency remarkably well with those theoretically predicted. Since the theory takes no account of dissipative effects, the heights of the experimental peaks are of course finite while the computed ones go to infinity. The experimental value of the cut-off frequency is about 2000 cycles, while for this frequency the theory still gives a small P_r'. However at 2100 cycles the theoretical curve goes practically to zero. The small theoretical peak just before the cut-off is difficult to understand, though it may be remarked that the same thing arises in the case of finite acoustic filters in air[2] (see particularly Fig. 2 in the paper referred to). The peak is so sharp that it may well have been missed in the experimental measurements. No transmission at all was observed in the range from 2000 to 12,000 cycles which agrees with the theoretical prediction that P_r' is negligible from 2100 cycles to 12,650 cycles. Here a narrow transmission band is indicated by the theory, extending from 12,650 cycles to 12,970 cycles. With the experimental arrangement used it would have been difficult to detect.

We wish to acknowledge with gratitude the assistance rendered in connection with the numerical calculations by Messrs. S. K. Shear, W. I. Fox and W. S. Cramer.

The Insulation of Air-Borne Sound

The Application of Laboratory Results to Practical Building Problems

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(Received March 12, 1938)

THE value of structural elements, walls, floors, windows, etc. for the insulation of air-borne sound is measured in the laboratory as a series of sound reduction factors at a number of frequencies. The sound reduction factor expresses the extent to which the *intensity* of the incident sound is reduced after transmission through an element of the particular construction under test. This is not, however, the property in which the designer or occupier of a building is directly interested. The designer wishes to know the extent to which the *loudness* of a sound is reduced after transmission through a *particular* element of construction such as a wall. This can be deduced at any one frequency from the reduction factor if allowance is made for the area of the wall, the absorption of the room into which the sound is transmitted (the "receiving" room) and the complex relationship between the objective intensity unit (decibel) and the subjective unit of equivalent loudness (phon). Such a deduction is much too complicated to be made in each instance and, moreover, it would strictly be necessary to know the frequency components of the sound to be dealt with in each case and to make the deduction at the appropriate frequencies. Apart from the fact that an analysis of the sound is seldom available—and indeed walls in buildings are usually at one time or another called upon to deal with all kinds of noise—there are obvious advantages in being able to ascribe a single figure to each form of construction. Frequently the arithmetic mean of the sound reduction factors taken over a representative range, say 200–2000 cycles per sec. is taken as an indication of the general insulating value of the wall and it is proposed to examine the validity of using such a figure as representing the reduction in equivalent loudness afforded to sounds of different frequencies.

Assuming that the significant component of the sound is transmitted into a room by the vibration of the wall separating the room from the source of sound, which is usually the case, then the reduction in intensity after transmission into the room is given very closely by the expression

$$10 \log_{10} (a/A\tau),$$

where a = total absorption of receiving room in sabins at the frequency under consideration,
A = area of dividing wall in sq. ft.,
$10 \log_{10} (1/\tau)$ = reduction factor of wall construction in decibels at the frequency under consideration.

Since the wall area and absorption of the receiving room come into this expression it is convenient to adopt a standard area and standard absorption. For these standards an area of 100 sq. ft. and an absorption of 100 sabins at 500 cycles per sec. are not untypical and are convenient.

In order to investigate the reduction in loudness afforded under these standard conditions it is necessary, taking into account the variation with frequency of reduction factors and absorption, to examine first the reduction in intensity at various frequencies due to various constructions and then to translate these intensities into units of equivalent loudness.

Neglecting the effects of resonance the reduction factors of very many normal constructions vary approximately linearly with frequency ratio, increasing very roughly by 5 decibels per octave[1] and the arithmetic mean of the reduction factors taken over a representative range of frequencies from 200–2000 cycles per sec. is approximately equal to the smoothed value of the reduction factor at 700 cycles. The absorption coefficients of materials do not vary in any regular manner

[1] This does not apply to certain forms of light complex construction such as double windows and some stud partitions, where the variation with frequency may be greater than 5 decibels per octave.

TABLE I. *Analysis for wall having average sound reduction factor = 60 db.*

Frequency cycles/sec.	Reduction Factor of Construction db	Absorption of Receiving Room sabins	Intensity Reduction after Transmission $10 \log_{10} (a/A\tau)$ db	Reduction in Equivalent Loudness (phons) of Sound of Original Loudness				Frequency cycles/sec.
				100 phons	90 phons	80 phons	70 phons	
125	48	50	45	66	72	72	>70	125
250	53	75	52	61	68	68	68	250
500	58	100	58	61	61	62	63	500
700	60	112	*60*	60	61	62	62	700
1000	63	125	64	64	64	64	64	1000

TABLE II. *Analysis for wall having average sound reduction factor = 50 db.*

Frequency cycles/sec.	Reduction Factor of Construction db	Absorption of Receiving Room sabins	Intensity Reduction after Transmission $10 \log_{10} (a/A\tau)$ db	Reduction in Equivalent Loudness (phons) of Sound of Original Loudness					Frequency cycles/sec.
				100 phons	90 phons	80 phons	70 phons	60 phons	
125	38	50	35	49	56	60	58	>60	125
250	43	75	42	48	51	56	55	55	250
500	48	100	48	48	50	52	53	52	500
700	50	112	*50*	49	50	52	52	52	700
1000	53	125	54	54	54	54	54	54	1000

TABLE III. *Analysis for wall having average sound reduction factor = 40 db.*

Frequency cycles/sec.	Reduction Factor of Construction db	Absorption of Receiving Room sabins	Intensity Reduction after Transmission $10 \log_{10} (a/A\tau)$ db	Reduction in Equivalent Loudness (phons) of Sound of Original Loudness					Frequency cycles/sec.
				100 phons	90 phons	80 phons	70 phons	60 phons	
125	28	50	25	31	39	42	42	44	125
250	33	75	32	35	39	40	43	43	250
500	38	100	38	37	40	40	41	41	500
700	40	112	*40*	40	40	40	41	42	700
1000	43	125	44	44	44	44	44	44	1000

TABLE IV. *Analysis for wall having average sound reduction factor = 30 db.*

Frequency cycles/sec.	Reduction Factor of Construction db	Absorption of Receiving Room sabins	Intensity Reduction after Transmission $10 \log_{10} (a/A\tau)$ db	Reduction in Equivalent Loudness (phons) of Sound of Original Loudness						Frequency cycles/sec.
				100 phons	90 phons	80 phons	70 phons	60 phons	50 phons	
125	18	50	15	16	21	25	26	27	27	125
250	23	75	22	23	26	28	30	31	31	250
500	28	100	28	27	29	30	30	31	31	500
700	30	112	*30*	30	30	30	30	31	31	700
1000	33	125	34	34	34	34	34	34	34	1000

TABLE V. *Analysis for wall having average sound reduction factor = 25 db.*

Frequency cycles/sec.	Reduction Factor of Construction db	Absorption of Receiving Room sabins	Intensity Reduction after Transmission $10 \log_{10} (a/A\tau)$ db	Reduction in Equilavent Loudness (phons) of Sound of Original Loudness						Frequency cycles/sec.
				100 phons	90 phons	80 phons	70 phons	60 phons	50 phons	
125	13	50	10	10	13	16	18	19	19	125
250	18	75	17	17	19	20	22	24	23	250
500	23	100	23	22	23	24	25	26	28	500
700	25	112	*25*	24	25	25	25	26	27	700
1000	28	125	29	29	29	29	29	29	29	1000

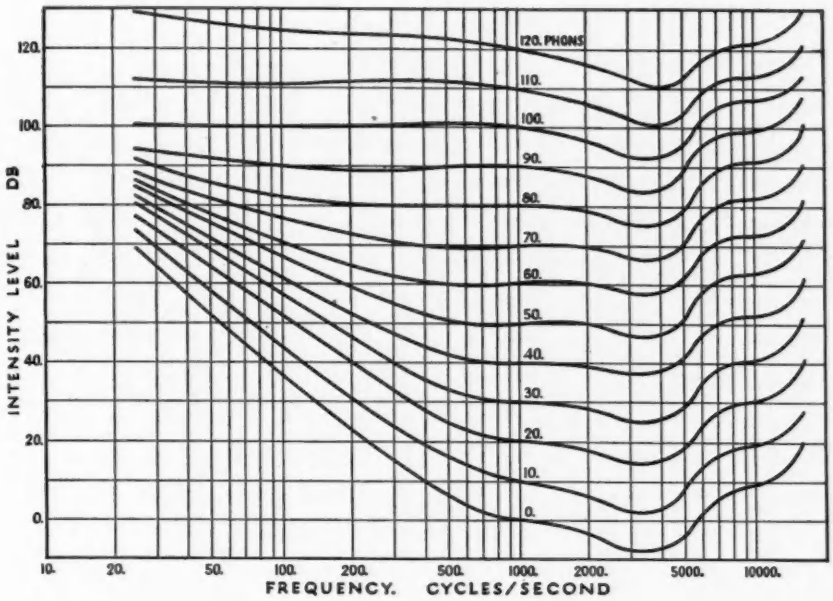

FIG. 1. Contours of equivalent loudness.

with frequency but it will be sufficiently accurate to assume the variation of total absorption of the standard room to be as follows:

Frequency, cycles/sec.	125	250	500	700	1000
Absorption, sabins.	50	75	100	112	125

The full analyses for constructions having five different series of reduction factors, each considered at five frequencies, are set out in Tables I–V. The translation from intensity units (db) to equivalent loudness units (phons) is based on the curves of equal loudness put forward by Fletcher and Munson which have been adopted in "American Tentative Standards for Noise Measurement, Z. 24.2—1936" and are reproduced in Fig. 1. The analysis is based on a standard area of wall of 100 sq. ft.

The figures set out show wide variations in the loudness reduction of any one partition, particularly in the case of partitions with low reduction factors. Even though small differences, say up to 4 or 5 phons, have no practical significance, the variations are so great that no single figure could possibly be taken as representative of any one wall in all cases and the variations would be greater if higher frequencies had been considered in the analysis, since the increase in the reduction factor is sufficient to ensure that loudness reductions at all higher frequencies exceed those at 1000 cycles. This, of course, is not surprising but the importance of having a single figure for approximate design purposes is very great and a figure representing the minimum extent to which loudness is reduced by a given wall is of considerable value. Even if regarded as representing the minimum reduction in equivalent loudness it will be seen that a figure equal to the average sound reduction factor is strictly not equally representative for highly insulating and poorly insulating walls alike. For instance, the reduction in loudness afforded by a wall whose average reduction factor is 50 db (Table II) of a sound of 90 phons at 125~ is 56 phons or 6 phons greater than the assumed figure. The loudness reduction of a similar sound by a wall whose average reduction factor is 25 db (Table V) is, however, 13 phons, or 12 phons less than the assumed figure.

At first sight, therefore, it appears that the "average sound reduction factor" is not a sound basis even for approximate design purposes. However, in practice by far the most common problem is to find a wall construction which will reduce sounds created on one side to inaudibility in the room on the other side. According to the level of masking noise this will represent a reduction to some loudness between, say, 0 and 40 phons. A wall with an average reduction factor of 30 db will therefore seldom be called

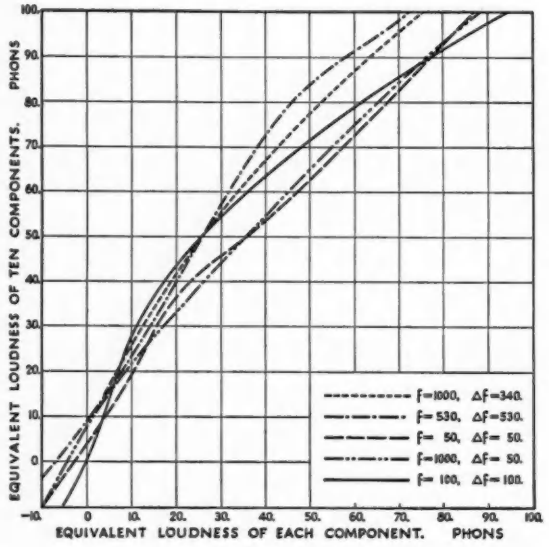

FIG. 2. Equivalent loudness of complex tones having ten equally loud components.

upon to deal with noises louder than, say, 70 phons and it will be seen from Table IV that for sounds not louder than 70 phons and of any of the frequencies examined, the average reduction factor basis does not involve any significant error in the case of this wall. Similar considerations apply to all walls having higher reduction factors and even in the case of the wall in Table V the error which arises at low frequencies is not very serious. The error would increase for constructions having even lower reduction factors than that in Table V but such constructions rarely, if ever, occur in buildings.

As far as pure tones are concerned, therefore, it is reasonable to adopt the average sound reduction factor as a basis for a first approximation of the minimum loudness reduction afforded by the construction. In practice, however, complex noises have to be dealt with. Fletcher and Munson[2] have made subjective measurements of the reduction in loudness of complex sounds made up of ten equally loud components, the results of which are reproduced here in Fig. 2, in which f refers to the frequency of the lowest pitched component and Δf to the difference in frequency between successive components. Assuming, on the basis of the foregoing discussion, that the loudness of each component will be reduced equally by a wall interposed in the path of the sound, these curves may be used to obtain an estimate of the reduction in loudness afforded by the wall to the combined sounds. Five types of complex sounds have been examined in this way and the results are set out in Tables VI–X.

[2] Harvey Fletcher and W. A. Munson, "Loudness, Its Definition, Measurement and Calculation," J. Acous. Soc. Am. 5, 282 (1933).

TABLE VI. *Reduction in loudness of complex sounds composed of ten equally loud components of frequencies 1000, 1340, 1680···(+340)···4060 cycles per sec.*

Equivalent Loudness of Complex Sound of Ten Equally Loud Components Phons	Reduction in Equivalent Loudness (Phons) of Complex Sound if Each Component is Reduced by				
	60	50	40	30	20
100	66	50			
90	75	57	43		
80	>80	67	48	35	
70	>70	>70	60	42	26
60	>60	>60	>60	45	30

TABLE VII. *Reduction in loudness of complex sounds composed of ten equally loud components of frequencies 530, 1060···(+530)···5300 cycles per sec.*

Equivalent Loudness of Complex Sound of Ten Equally Loud Components Phons	Reduction in Equivalent Loudness (Phons) of Complex Sound if Each Component is Reduced by				
	60	50	40	30	20
100	75	57			
90	82	70	53		
80	>80	75	63	48	
70	>70	>70	63	50	33
60	>60	>60	>60	48	35

From Tables VI and VII it will be seen that the complex sounds of high frequency are reduced to a greater extent than each component so that even on the assumption that each component is equally reduced the average reduction factor basis underestimates the reduction afforded by a wall to these particular sounds. Actually the wall would afford greater reduction to the higher frequency components so that the reduction of the complex sound would in fact presumably be greater still. The error involved is thus on the "safe" side from the designers' point of view.

For the complex sounds examined in Tables VIII and IX, in which the frequencies of neigh-

boring components differ by only 30 cycles per sec., the reduction of the complex sounds do not differ significantly from the reduction of each component and the average reduction factor basis may be at once accepted for these sounds.

From Table X it will be seen that the reduction of this particular sound is in some cases markedly less and in other cases considerably greater than the reduction of each component. For this sound, therefore, the average reduction factor basis may sometimes significantly overestimate the insulating value of the wall.

It is obviously difficult to draw from the above data on sounds made up of equally loud components conclusions applicable to all the variety of complex sounds likely to occur in buildings. It would seem reasonable, however, to suppose that the method of estimating the reduction in equivalent loudness afforded by a wall on the basis of its average sound reduction factor is more likely to prove an underestimate than an overestimate, so that as an estimate of the minimum loudness reduction it forms, though far from an ideal, at least a reasonable basis for approximate work.

Strictly, this holds only where the area of the wall, in square feet, is numerically equal to the absorption of the receiving room, in sabins, and allowance should theoretically be made in each case for the area of the particular wall and for the absorption of the particular receiving room in question. In practice, however, the value of $10 \log_{10} (a/A)$ would seldom fall outside the range ± 5, so that in the majority of cases the effects of area and absorption may be neglected.

TABLE VIII. *Reduction in loudness of complex sounds composed of ten equally loud components of frequencies 1000, 1050, 1100···(+50)···1450 cycles per sec.*

Equivalent Loudness of Complex Sound of Ten Equally Loud Components Phons	Reduction in Equivalent Loudness (Phons) of Complex Sound if Each Component is Reduced by				
	60	50	40	30	20
100	57	48			
90	61	50	39		
80	64	52	41	31	
70	70	54	42	31	21
60	>60	60	44	32	21

TABLE IX. *Reduction in loudness of complex sounds composed of ten equally loud components of frequencies 50, 100···(+50)···500 cycles per sec.*

Equivalent Loudness of Complex Sound of Ten Equally Loud Components Phons	Reduction in Equivalent Loudness (Phons) of Complex Sound if Each Component is Reduced by				
	60	50	40	30	20
100	57	50			
90	58	47	40		
80	65	48	37	30	
70	70	56	38	27	22
60	>60	60	45	28	17

TABLE X. *Reduction in loudness of complex sounds composed of ten equally loud components of frequencies 100, 200···(+100)···1000 cycles per sec.*

Equivalent Loudness of Complex Sound of Ten Equally Loud Components Phons	Reduction in Equivalent Loudness (Phons) of Complex Sound if Each Component is Reduced by				
	60	50	40	30	20
100	42	34			
90	50	38	29		
80	75	50	35	24	
70	>70	>70	47	29	18
60	>60	>60	60	44	22

Conclusions

The sound reduction factors at various frequencies of an element of construction have little significance (except as affording comparisons between different constructions) to the architect or builder who is interested in the loudness and not in the intensity of sounds.

A figure which may be derived by taking the arithmetic mean of a representative number of sound reduction factors over the frequency range 200–2000 cycles per sec. does, however, give an approximation to the minimum reduction in equivalent loudness afforded, under practical conditions, by many walls to many sounds. This average reduction factor provides, therefore, a useful indication to the architect of the general efficiency of a wall as insulation against the direct transmission of air-borne sound.

Acoustical Output of Air Sound Senders*

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(Received November 23, 1937)

The use of the Rayleigh disk for measurement of extremely great sound intensities is discussed, and it is stated that the Rayleigh disk method is applicable also in this case. The Rayleigh disk may even be used in the open air as part of a pointer instrument when properly shielded, thus giving a means of directly measuring the actual sound intensity (in watts/cm²) of intense sound fields. The measurement also involves a determination of the direction of propagation of energy. The calculation of the total flux over the opening of a sound sender, giving the actual acoustical output in watts, is in that way very much simplified. As a secondary instrument a crystal microphone, calibrated by comparison with the Rayleigh disk instrument, was used. When used in extreme sound fields it was directly connected with a rectifier galvanometer. Reports are given of measurements of the total output of a membrane sender and of that of a diaphone. In the first case measurements were also made of the different kinds of losses occurring both in the electrical and the acoustical oscillation circuit, while the vibration losses were determined from the power balance and vibration measurements. Oscillograms of sound wave form are given and discussed.

1. INTRODUCTION

AT the end of 1935 the Norwegian Lighthouse Authorities asked this institute to measure the actual output under working conditions of two types of air sound senders for fog signals, *viz.* a membrane sender and a diaphone. We first made some preliminary studies of the sound field of the membrane sender, using a condenser microphone in the near field and an auditory comparison method in the distant field. It was obvious that even the near field was subject to peculiar irregularities not only in space, but also in time. Consequently the field flux giving the total output had to be measured through a surface at the very opening of the sender. The conditions existing in the sound field are here, however, considerably different from those ordinarily met with in laboratory experiments, the sound intensity at the opening of a normal sound sender for fog signals being of the order of size of 150 db. First of all we, therefore, had to work out a convenient method for measuring the sound intensity at great intensities and apply this to determine the total sound flux. The report of this part of our work is given here.[1] Secondly we have started an experimental study of the structure of the field in order to investigate the peculiar irregularities mentioned above, which according to our opinion reveal an effect of turbulence upon the propagation of sound that is generally neglected.[2]

2. ABSOLUTE MEASUREMENT OF SOUND AT GREAT INTENSITIES

To perform an absolute measurement of the intensity of sound, we shall have to measure the r.m.s. value of *either* the sound pressure p, *or* the velocity s of the air particles, *or* the amplitude A of oscillation of the air particles, according to the equations

$$J=\frac{p^2}{c\rho}=s^2c\rho=4\pi^2f^2A^2c\rho \quad \text{erg}\cdot\text{cm}^{-2}\,\text{sec.}^{-1}, \quad (2.1)$$

where c is the velocity of propagation of sound and ρ the density (at normal conditions $c\rho=41.5$ c.g.s. for air).

It is assumed here that no phase difference exists between the sound pressure, p, and the velocity of the air particles, s, or that the difference is practically negligible. In the cases treated by us we traced the flux lines at the opening of the foghorn (Section 3) and found from calculations based upon the curvature of the flux lines that the phase difference was

* Publikasjoner fra Chr. Michelsens Institutt No. 95.

[1] A preliminary report was presented at the 1936 Chr. Michelsen's Institute Annual Meeting, see Olaf Devik, Beretninger VI, 2, 1936, Chr. Michelsens Institutt, Bergen.

[2] See note by the authors in Nature, **139**, 550 (1937). Some recent results were presented by Olaf Devik at the Berlin International Lighthouse Conference 1937.

negligible for our purpose, which was to determine the total flux of energy within a tolerance of a few percent.

The measurement of sound pressure, p, is in practice mostly performed either with a condenser microphone or with a crystal microphone. In most cases where the audibility of sound is concerned the measurement of sound pressure within a tolerance of 1 decibel (i.e., of 24 percent of the intensity) is quite sufficient, and in such cases the condenser microphone is often calibrated by an electrostatic method. Even if this method might be used at the very high sound pressures here concerned (10^3–10^4 dyne cm^{-2}), the accuracy would scarcely be sufficient for the determination of the output where a tolerance of only a few percent is wanted. In that case the calibration of either a condenser microphone or a crystal microphone by comparison with a Rayleigh disk instrument presents itself as the most promising method. We shall, however, have to discuss it to some detail in the following, as the application of the Rayleigh disk in extreme intense sound fields may encounter certain difficulties, as already pointed out by Wente.[3]

In principle the Rayleigh disk method represents a measurement of the velocity, s, of the air particles. A method based upon the measurement of the third quantity mentioned above, the amplitude A of the air particles, is at present scarcely practicable for sound fields in the open air, although good results have been obtained by Andrade and Parker[4] by registering optically the amplitude of suspended smoke particles in resonating tubes. Another way of performing absolute measurements is to calibrate the microphone in a standard sound field, the intensity of which is directly calculable. In laboratories there is often used a piston chamber to establish a sound pressure of known intensity. It is, however, difficult to eliminate perturbing vibrations of the walls; it ought to be remembered that pressure variations of an intensity say 90 db of a sound of 300 cycles per second might be practically compensated if the housing vibrates with an amplitude of only one micron, as an anti-phase correspondence between pressure variation and chamber oscillation will regularly occur in this case.

This objection is not valid with regard to the method applied by Zernov,[5] who tested the application of the Rayleigh disk. He employed a vibrating cylindrical chamber, and into this was lowered the Rayleigh disk, suspended by a torsion fiber fastened at a fixed point. The vibrations of the chamber are then equal to the displacement of the air particles in the chamber with respect to the Rayleigh disk, and provided that the dimension of the chamber in the direction of oscillation is small compared with the wave-length corresponding to the frequency, the r.m.s. kinetic energy will be equivalent to that of a progressive passing wave of the intensity

$$J = 4\pi^2 f^2 A^2 c\rho \quad \text{erg}\cdot\text{cm}^{-2}\,\text{sec.}^{-1}, \qquad (2.2)$$

A being the r.m.s. amplitude of the oscillating air particles. A considerable advantage of the Zernov method is that the energy actually radiated by the cylinder is very small, and thus the vibrations may be maintained simply by an electrically driven sound fork. In that way Zernov was able to apply amplitudes of oscillation up to 0.75 millimetres (the r.m.s. amplitude A thus being 0.053 cm) and obtained an equivalent sound field intensity $J = 3.8 \cdot 10^4$ erg$\cdot$cm^{-2}sec.$^{-1}$, which would correspond to 136 db. From the point of view here considered it must be emphasized that the experiment of Zernov was thus performed under conditions equivalent to sound fields of great intensities.

The sound intensity according to Eq. (2.2) was compared by Zernov with the value calculated from the torque which the Rayleigh disk is exposed to. According to Rayleigh and to König[6] the couple exercised upon an extremely thin disk, of radius r and placed at an angle α with the direction of sound propagation, is

$$M = \frac{4}{3}\rho r^3 s^2 \sin 2\alpha \quad \text{dyne cm.} \qquad (2.3)$$

Introducing Eq. (2.1) the sound intensity as calculated from the Rayleigh disk method will be

$$J = \frac{3}{4}\frac{cM}{r^3 \sin 2\alpha} \quad \text{erg}\cdot\text{cm}^{-2}\,\text{sec.}^{-1}. \qquad (2.4)$$

[3] E. C. Wente, J. Acous. Soc. Am. **7**, 1 (1935).
[4] E. N. da C. Andrade and R. S. Parker, Proc. Roy. Soc. **159**, 507 (1937).
[5] W. Zernov, Ann. d. Physik **26**, 79 (1908).
[6] W. König, Ann. d. Physik **43**, 43 (1891).

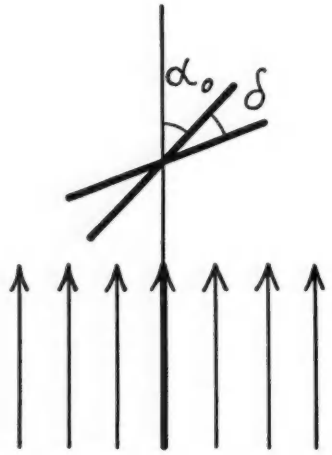

FIG. 1.

This formula is directly applicable when a compensating couple is applied, equal and opposite to the couple M exercised by the field, the Rayleigh disk deflected by the field thereby returning to its equilibrium position. It is seen from the formula (2.4) that the sensitivity of observation is greatest when the angle α is 45°, while it is rapidly decreasing as the angle approaches 90°.

When compensation is not used the Rayleigh disk will be deflected from its equilibrium position (α_0) until the couple exercised by the field is balanced by the couple due to the torsion of the suspension, the angle then being $\alpha=\alpha_0+\delta$ (Fig. 1). If we put $M=k\delta$, k being determined by calibration, the sound intensity will be

$$J=\frac{3}{4}\frac{ck\delta}{r^3\sin 2(\alpha_0+\delta)}\ \text{erg}\cdot\text{cm}^{-2}\,\text{sec.}^{-1}. \quad (2.5)$$

In this case the sensitivity of observation depends upon the angle δ, the radius r and the angle $(\alpha_0+\delta)$, the total sensitivity being greatest when the angle $(\alpha_0+\delta)$ is in the region of 45°. It will be seen that it is thus necessary to adjust the torsion of the suspension as well as the size of the Rayleigh disk to the actual sound field if maximum sensitivity of the measuring device is wanted.

The Rayleigh formula (2.3) presupposes that the radius r of the extremely thin disk is less than $\lambda/2\pi$, λ being the wave-length. This condition being fulfilled, the indications of the Rayleigh disk are, according to theory, independent of frequency. Zernov found full accordance between the actual vibration energy and that measured by the Rayleigh disk method, and he also determined experimentally the correction which had to be applied when the thickness of the disk ($2b$) was an appreciable fraction of the diameter ($2r$). From his correction formula follows that with $b/r=0.01$ the value of the sound intensity J given by Eqs. (2.4) and (2.5), respectively, is to be reduced by 2.7 percent.

Treating the theory of the Rayleigh disk in a recent paper L. V. King[7] states that Rayleigh's original formula (2.3) will have to be corrected, but the correction for finite thickness of the disk is practically inappreciable for disks in air. If we calculate the correction for a mica disk of radius $r=1$ cm and $b/r=0.01$, according to the formula given by King, we find that the value of J given by Eqs. (2.4) and (2.5), respectively, is to be reduced by 1.8 percent, this correction thus being a little smaller than that found empirically by Zernov in the same case, as mentioned above. We may thus conclude that the formulae (2.4) and (2.5), respectively, will give the correct value within 1–2 percent when a thin disk is used, the diameter of which is small compared with the wave-length in question.

There is, however, still one question to consider. The theoretical deduction of the Rayleigh formula (2.3) by König[6] and King[7] is based upon the assumption that the velocity of the air in every point may be derived from a velocity potential, i.e., that the motion of the air particles is irrotational. If, however, the velocity is so great that eddies are formed at the circumference of the Rayleigh disk, this assumption would be no more valid. We have noticed that according to Zernov the Rayleigh disk gives correct results under conditions equivalent to the fairly great intensity of 136 db; it might, however, be objected that Zernov applied only one frequency, *viz.* 91.5 c/sec. and a test at other frequencies might seem desirable. From a theoretical point of view we do not consider the existence of eddies in this case very probable, because a forced oscillation causing relatively small coordinated displacements of the air particles may be assumed to act very efficiently in smoothing out all irregularities with respect to phase, thus counter-

[7] L. V. King, Proc. Roy. Soc. **153**, 1 (1935).

acting the formation of eddies, so well known in progressive currents. Considering the importance of the question, however, we established an independent test. The influence of eddies eventually being formed at the circumference of the Rayleigh disk in a sound field of constant intensity would probably depend on the length of the circumference, i.e., of the first power of the radius of the disk, while the pure Rayleigh effect is proportional to r^3. A variation of the radius r within a great range might therefore be expected to give very different values of the intensity calculated, if eddy disturbances really occurred. Accordingly we made measurements with Rayleigh disks of the different sizes $r=1.0$, 1.5 and 10.5 cm, in the sound field of a 300 c/sec. sender (Section 6), the results of which are reproduced in the Fig. 2. It must be noticed that the observations here recorded are not quite homogeneous with respect to tolerance. This is due to practical difficulties: first, the observation series were taken in the open air at the lighthouse, exposed to differing weather conditions; secondly, we had for the two small disks the same torsion instrument, the sensitivity being accordingly considerably smaller for the smallest disk than for the normal disk of $r=1.5$ cm, the size of which was adjusted to the torsion of the suspension as well as to the intensity of the field, as discussed above; thirdly, the instrument with the largest disk had only a moderate sensitivity as it was made in the workshop of the lighthouse during our visit there.

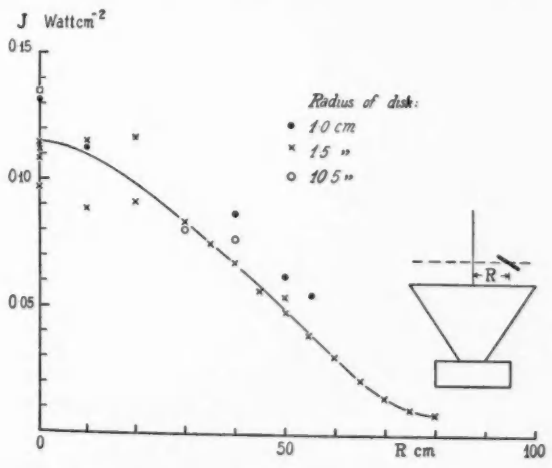

FIG. 2. Rayleigh disk measurements in front of membrane sound sender (distance 16 cm), plotted to the distance R from the central axis. Intensity in watts/cm².

Taking into account the possible individual errors of the different observations and considering especially the series for $r=1.5$ cm, there is no difficulty in drawing an intensity curve as that given in the Fig. 2, and we accordingly conclude that the results give no evidence of turbulent motion caused by the disk.[8]

This test thus confirms the experiments of *Zernov: the absolute method of the Rayleigh disk is applicable at the greatest intensities produced by air sound senders in practice*, at any rate within a tolerance of a few percent.

3. Rayleigh Disk Instrument for Great Sound Intensities

As is well known, the application of the Rayleigh disk in sound fields of moderate intensity is rather inconvenient on account of the fact that a torque is also caused by air currents. The formula (2.3) for the torque exercised upon the disk is in fact also valid for direct current flow, and we may consequently estimate the velocity of a permissible air current if the torque due to the sound field is not to be changed more than say one percent. As the torque is proportional to s^2 the air current needs only to be less than 1/10 of the r.m.s. velocity s in order that this condition may be fulfilled. In the Table I both s and the permissible air current calculated as 0.1 s are tabulated at different sound pressures and corresponding intensities: If we consider the values of the last column giving the permissible velocities of disturbing air currents, it is not astonishing that the Rayleigh disk is extremely sensible to air currents at sound intensities below 100 db.

[8] We observe from Fig. 2 that there is a comparatively great spreading of the observations near to the axis of the field even in the series of $r=1.5$ cm. We suppose that this fact is not due to fluctuations of the chief field (of frequency 300 c/sec.), but is caused by radiation from the wall of the cone. The cone will have a tendency to oscillate more or less regularly at harmonic frequencies, and a maximum effect will eventually be observed in the central parts of the opening, a convergence of the perturbing radiation being caused by the shape of the cone. According to these considerations we have drawn the intensity curve of the chief field of a regular shape with the maximum in the axis. It ought to be observed that the central part of the field contributes little to the total output on account of the small section, and it is therefore of minor importance if the intensity curve is drawn with some tolerance for the central part of the field.

On the other hand, however, it is equally evident that at very great sound intensities (140–150 db) the perturbing influence of a wind velocity of even some meters per second may be completely eliminated if a convenient shielding cage is applied, which reduces the velocity of the air within the cage to some centimeters per second. The reason is that the torque exercised upon the Rayleigh disk increases in the same proportion as the intensity of the field (see Eq. 2.4). It is seen from the Eq. (2.3) in connection with the Table I that a Rayleigh disk of radius say 1 cm will be submitted to a torque of about 100 dyne cm in a sound field of 154 db, and to 1 dyne cm, respectively, in a field of 134 db ($\alpha = 45°$). These values are ample for pointer instruments. As a consequence the analysis of the Rayleigh disk method led us to the design of *a Rayleigh-disk pointer instrument for use in the open air at great sound intensities*. The first instrument constructed by us had the disk simply attached to the axis of a moving coil galvanometer, as shown in the Fig. 3. The instrument was shielded by a spacious cylindrical double-walled net cage covered by gauze. This shielding proved to be efficient in a wind of 3–4 m/sec. If the thickness of the shielding layers was doubled no influence upon the measurements was noticeable, and this was also confirmed by experiments in the laboratory; for the low frequencies with which we are here concerned gauze layers are without influence upon the sound field. Fig. 9, below demonstrates how the instrument was placed at the lighthouse tower during the measurements performed in February, 1936.

It may be mentioned that the Rayleigh disk instrument may even be used at the opening of a diaphone, in spite of the air blast, provided that the shielding be very careful. In view of the complexity of the sound spectrum of a diaphone (see Section 11) the Rayleigh disk as an absolute

FIG. 3. First type of Rayleigh disk pointer instrument. The shielding cage is removed and the mica disk is replaced by a black paper disk, in order to get a better photography.

instrument has the great advantage of being independent of frequency. In the actual case, of which a description is given in Section 9 below, we found it, however, more convenient to use the calibrated crystal microphone for the measurement of intensity, the Rayleigh disk being used only for the determination of the direction of sound. This was done also because of some imperfection of the instrument, which was not so strictly symmetrical in construction as the instrument described above (Fig. 3) and used in the measurements on the membrane sender. From these experiences it became obvious that the Rayleigh disk instrument must be constructed with cylindrical symmetry with respect to the axis of torsion of the disk itself, and it is especially important that there are no supporting rods or similar obstructions near the disk. A technical form of the instrument, of the type shown in Fig. 3, is being developed at this institute by Mr. Odd Dahl, and details of construction will be given later.

In a sound field the application of the Rayleigh disk requires knowledge of the flux lines which represent the direction of sound propagation in every point. This direction is easily observed by means of the Rayleigh disk itself, as the couple exercised upon the disk is zero when the disk is

TABLE I.

r.m.s. SOUND PRESSURE dyne/cm²	INTENSITY db	r.m.s. VELOCITY s cm/sec.	PERMISSIBLE AIR CURRENT cm/sec.
1	73.8	0.024	0.0024
10	93.8	0.24	0.024
100	113.8	2.4	0.24
1000	133.8	24	2.4
10000	153.8	240	24

perpendicular to the sound direction. As an illustration we quote the case represented by Fig. 9. The position of the disk with respect to the instrument scale was here indicated by the pointer attached to the disk, and the position of the instrument with respect to a fixed azimuth scale was indicated by a pointer attached to the instrument. For direction finding the instrument is turned round its axis until the disk remains at the equilibrium position during the sending period, the torque of the sound field on the disk being zero. The direction of sound is then perpendicular to the disk, and the corresponding reading of the instrument position on the azimuth scale is observed. In Fig. 4 the direction observations in an actual case are given. By means of these observations a flux line chart of the field may be drawn, as illustrated in Fig. 7.

When the sound direction at the point considered is observed in this manner, the instrument is turned a definite angle (30° or 45° are usually used) on the azimuth scale, counted from the sound direction. During the sound sending the Rayleigh disk, which is now at an angle α_0 with the direction of propagation, will be deflected an angle δ, which is indicated by the pointer on the instrument scale. The intensity of sound will then be given by Eq. (2.5). The direction of sound as well as the intensity are in this way measured by the same instrument in two subsequent operations.

For the determination of the direction only it might of course be possible to use a Rayleigh disk pivoted without torsion so that it could move freely around its axis. This would permit observation of the sound direction immediately, as the disk would place itself perpendicular to the sound direction. Such a device, properly damped, might be of importance when a secondary instrument is to be used for the measurement of the intensity as proposed in the following section, and it might be useful for a qualitative study of a strong sound field as well.

4. The Crystal Microphone Applied at Great Sound Intensities

The crystal microphone when calibrated by comparison with a Rayleigh disk instrument at great intensities is probably at present the most convenient instrument for the measurements of strong sound fields, as it may be *connected directly with a rectifier galvanometer* of proper sensitivity. The crystal microphone then acts as an energy transformer and the application of an amplifier is superfluous in strong sound fields. On the other hand, connected with a convenient amplifier, the same crystal microphone may be used for the measurement of very weak sound fields. The instrumental equipment necessary both for the measurement of sound intensity in the extended field and for the measurement of the intensity at the very opening of the sender, which is required for the calculation of the output, is thus much simplified by the use of the crystal microphone. The calibration by the Rayleigh disk may be performed even in the open air at great sound intensities, as described above. Reference is given to Section 9, where measurements with a crystal microphone of the sound field of a diaphone are treated.

5. Calculation of the Total Acoustical Output

In a progressive wave the flux of energy per second passing through a given surface is

$$S=\sum_F J\cos\beta\, dF, \qquad (5.1)$$

where β is the angle between the direction of propagation and the normal of the element dF. The intensity J is calculated from the Eq. (2.5) for a sufficient number of points in the field. The sound senders here concerned have conical horns.

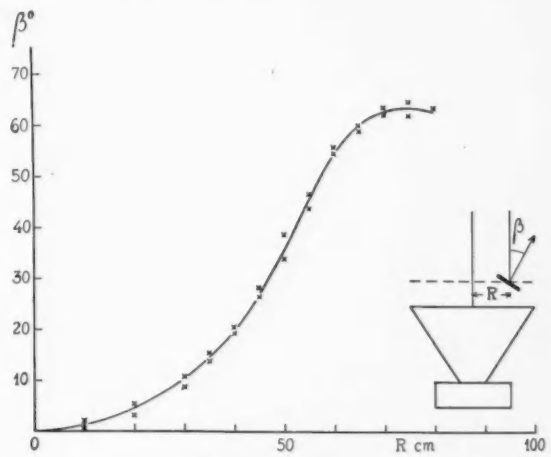

Fig. 4. Observations of sound direction in front of membrane sender with Rayleigh disk pointer instrument.

The sound field just outside the opening of the cone is, therefore, symmetrical with respect to the axis. It will then be sufficient to determine the sound intensity J and the direction of propagation for a number of points on a line situated in a plane of symmetry, this line being chosen in such a way as to give a representative section of the field. Having regard to the mounting of the instrument it will be practical either to move the instrument in a straight horizontal line normal to the axis of symmetry and sufficiently near to the opening of the cone, or to move it in a horizontal circular line surrounding the opening, with the center at a point of the axis inside the cone.

In the Figs. 2 and 4 are given the measurements of J and β, respectively, along a straight line 16 cms in the front of the cone of a membrane sender, plotted against the distance from the axis.

Choosing as element of area a sectional ring $dF=2\pi RdR$ we then calculate the quantity

$$dS/dR=2\pi RJ\cos\beta, \qquad (5.2)$$

which is represented in our example by Fig. 5. Then the flux of energy is determined by graphical integration of this curve

$$S_R=\int_0^R (dS/dR)dR, \qquad (5.3)$$

giving the flux through a circular section as

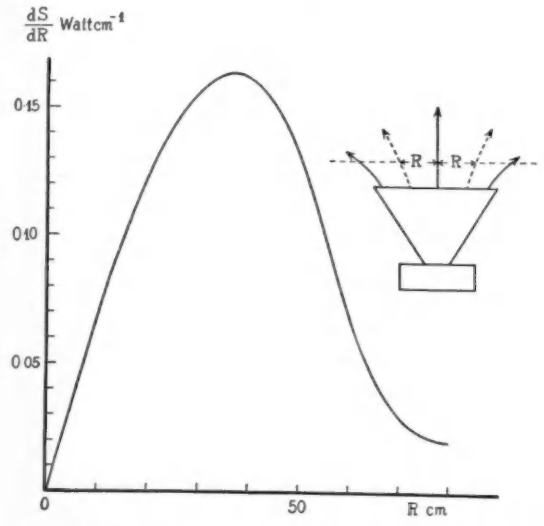

FIG. 5. Increase of energy flux by the increase of radius of the section, dS/dR, according to Eq. (5.2), for a membrane sound sender.

function of the radius. From the Fig. 6 representing S_R we notice that by increasing values of R the flux is approaching a limit S, which is easily found by extrapolation, in this case very nearly 800 watts. By this method of calculation it is not necessary to intersect the entire bundle of flux tubes, if only the extrapolation may be performed with sufficient approximation.

From the curve representing S_R of Fig. 6 we may observe the values of R corresponding to

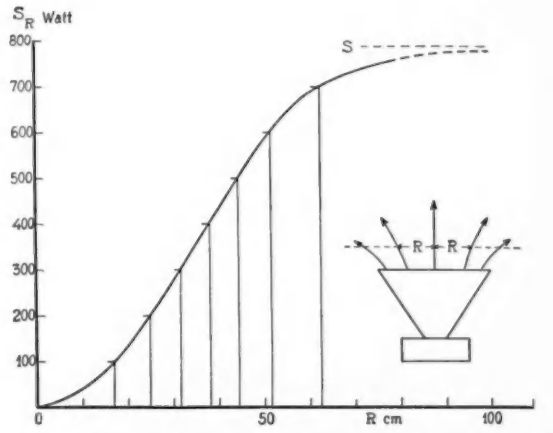

FIG. 6. Energy flux S_R, plotted to the radius of the field section, according to Eq. (5.3), for a membrane sound sender.

100, 200, $\cdots$ watts, as indicated in the figure, and use these together with the observed values of β (direction of propagation Fig. 4) to draw tubes carrying a flux of 100 watts each. Such tubes are represented in Fig. 7 for the membrane sender.

6. Acoustical Output of an Electromagnetic Membrane Sender

In this section a short report will be given of measurements performed under working conditions. The fog signal plant in question consists of two electromagnetic sound transmitters of the membrane type, which are operated at an acoustical frequency of 300 cycles, each transmitter being part of an electrical oscillation circuit tuned to resonance at 150 cycles. The two oscillation circuits (Fig. 8) are connected in parallel with the alternating generator during the signal period, while they are shortcircuited in the pause period, during which the electric energy generated is consumed in a resistance load. The

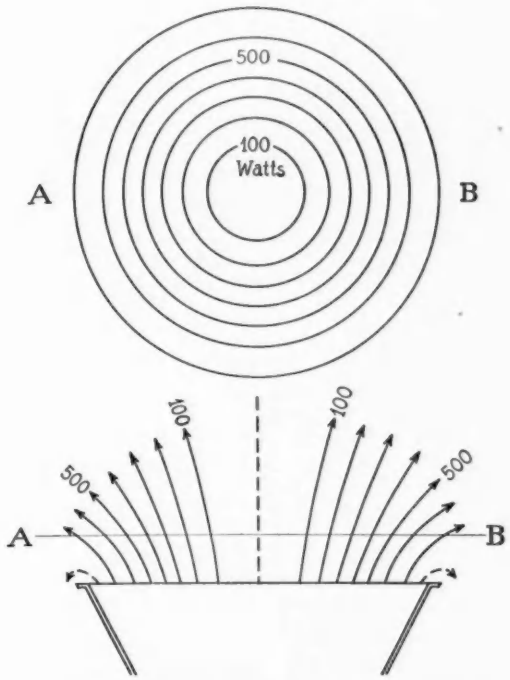

FIG. 7. Streamline tubes for a membrane sound sender, each tube carrying 100 watts (circular section).

electric input to each oscillation circuit is 4250 watts, of which 175 watts are consumed in a resistance which prevents the transients from rising dangerously at the switching moment.

The stand carrying the Rayleigh disk instrument was mounted on the balcony of the lighthouse tower (see Fig. 9) and could be moved in a direction perpendicular to the axis of the sound field, the Rayleigh disk thereby crossing the sound field in a straight line situated in a plane of symmetry. This line of observation was generally chosen 16 cm in front of the cone of the transmitter.

The measurements of sound direction and sound intensity were chiefly made with a Rayleigh disk instrument of the type represented by Fig. 3. This instrument had electromagnetic damping, and the deflection of the pointer was observed with a telescope from the balcony of the lighthouse. At one series of observations another type of instrument was used, having the disk mounted on a thin horizontal shaft, which was pivoted with springs and equipped with reflecting mirror for observation at a distance; this instrument had an oil damping arrangement at the shaft end.

The observations made with different sizes of the Rayleigh disk have already been recorded on Fig. 2, on which also the average distribution of intensity is given by the curve drawn. The calculation of the total output has also been treated in the preceding section as a demonstration of the method of calculation and from Fig. 6 is seen that the flux S_r crossing a vertical section through the line of observation is approaching a limit S of nearly *800 watts*, which is thus *the total acoustical output of one transmitter*. The probable error of this measurement is, according to our estimate (see also reference 8), less than 10 percent.

7. Power Balance of the Membrane Sender

To complete the analysis of the transmitter the losses of energy in the different parts of the electrical and the acoustical oscillation circuit were measured. So were the vibrations transferred to the walls (Section 8), but as these measurements allow only an approximate estimation of the power lost by vibration, a more

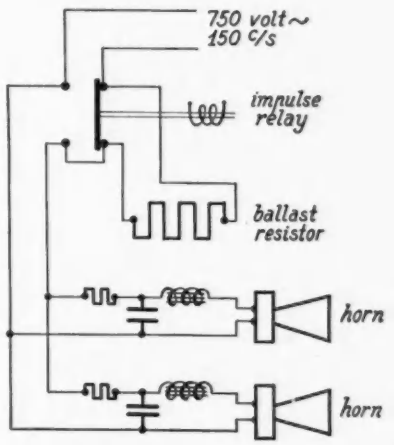

FIG. 8. Oscillation circuit of two membrane sound senders, operated in parallel.

accurate value is calculated by means of the power balance, which is given below and illustrated in Fig. 11.

The *losses in the transmitter itself* were calculated from temperature measurements performed during a sending period of 4.75 hours of normal operation of the transmitter, and the subsequent cooling down (during 4 hours). The heat production in the solid material of the transmitter during operation is chiefly due to magnetic

hysteresis in the iron core and the yoke of the magnetic circuit, as well as to elastic hysteresis in the vibrating steel membrane. The temperature distribution was determined first by thermojunctions placed at several representative points of the surface, secondly with a resistance thermometer consisting of a thin enamel-insulated copper wire wound in many turns around the circumference of the membrane housing, and thirdly the resistance of the transmitter coil was measured at the beginning and the end of the sending period, thereby giving the total temperature rise in the inner parts. The transfer of heat from the inner parts to the surface will cause the temperature of the surface to lag behind the average temperature of the transmitter. Thus, when the operation ceases the surface temperature continues to rise for a short while, as represented by Fig. 10.

The evaluation of the heat production in this case being treated in a separate paper[9] we here quote the result only, which was 1.94 kw-hr. during the operation period of 4.75 hours. In order to calculate the corresponding power loss it must be noticed that the net sending time is only $\frac{1}{5}$ of the operating period, hence the power loss is $1.94 \cdot 5/4.75 = 2.04$ kw.

There are, however, some minor losses in the transmitter itself that are not accounted for above. In the air volume contained in the resonance chamber and the communicating cone some energy is dissipated. Part of this will be transferred to the walls of the chamber and enter into the heat losses calculated above, but some heated air will escape during each sending period through a draining tube at the bottom of the resonance chamber and the air within the cone will transfer almost all the heat developed in it to the open air or to the cone, which has about the same effect. The temperatures measured at proper places in this air volume show a rapid rise during each sending, followed by a slow sinking during the following pause. From the temperature changes and the air masses involved we calculate the amount of power lost in this manner to be about 170 watts.

The losses in the transmitter itself are thus in all $2040+170=2210 \sim$ *2200 watts*. The probable error of this value may be about 5 percent.

[9] Chr. Michelsens Institutt, Beretninger VIII, 2.

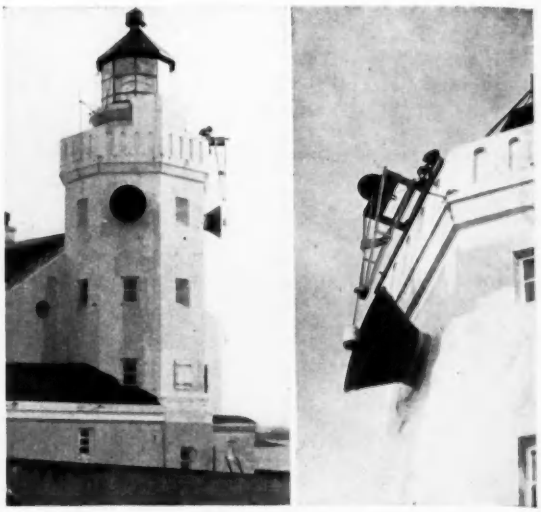

FIG. 9. Mounting of Rayleigh disk instrument at a membrane sound sender.

The other losses in the oscillation circuit are calculated from resistance measurements of the cables and from temperature measurements of the condenser and the induction coil. The values are recorded in the following *power balance:*

A. Electrical input at the terminals of the oscillation circuit:		4250 watts
B. Power loss:		
Transmitter	2200 watts	
Condenser	55	
Induction coil	100	
Cables	220	
Resistance load	175	
Vibration losses	(700)	
C. Acoustical output:	800 watts	
	Total	4250 watts

The power balance is represented by Fig. 11. The vibration losses are calculated as the difference between the input and the recorded losses plus the acoustical output. The limit of error of this indirect measurement is probably about 20 percent. From the power balance recorded we realize that the ratio output/input for the transmitter circuit is $800/4250=0.188$ while the mechanical-electrical efficiency of the electromagnetic membrane sender itself is $800/3000=0.266$, i.e., 27 percent.

8. VIBRATION MEASUREMENTS

In view of the fairly considerable value calculated for the vibration losses in the preceding

section, measurements were performed (June, 1937) with a crystal vibration pick-up connected with an amplifier, the set being calibrated to give absolute measurements of the amplitude of oscillation. Each transmitter of the plant considered is mounted on an iron frame rigidly connected with the wall of the lighthouse, the transmitter being placed with horizontal axis in a conical window opening (see Fig. 9 where the cones of the transmitters are seen from outside). The thickness of the wall is 0.6 m. The vibrations transferred by the frame cause elastic waves in the volume of the wall to radiate as if the source were in the conical opening. In fact, at a circumference described with a radius of about one meter and centered in the opening, the amplitude of oscillation perpendicular to the wall's surface has approximately a constant value (of $3.6 \cdot 10^{-4}$ cm), and there is a pronounced attenuation as we proceed downwards in the tower, when the local irregularities of the curve caused by the structure of the tower are smoothed out. Picturing the radiation as diverging chiefly from the center of the opening, we do not, however, know the partition of the radiant energy between the transverse and longitudinal oscillations, and only to get an idea of the order of magnitude of the energy flux we calculate it as if it were a progressive cylindrical wave with amplitude within the limits $1.8 \cdot 10^{-4}$ and $3.6 \cdot 10^{-4}$ cm, the first value being chosen as half of the measured one to account for the total reflection at the surface of the wall. Applying Eq. (2.1), $c\rho$ now being referred to the material of the wall ($=1.1 \cdot 10^6$ g cm^{-2} sec.$^{-1}$), the intensity corresponding to the amplitude range will be $J=0.0125$ and 0.05 watts/cm^2, respectively. In the idealized case of a cylindrical wave passing through a section of diameter 2 m and height 0.6 m, the corresponding total flux would be 470 and 1880 watts, respectively. The real energy consumption due to the complicated radiation will probably correspond more nearly with the first estimate of the amplitude where total reflection is accounted for, than with the second, a vibration loss somewhere between a half and one kilowatt should thus be a reasonable estimate, confirming in order of magnitude the value calculated above from the power balance.

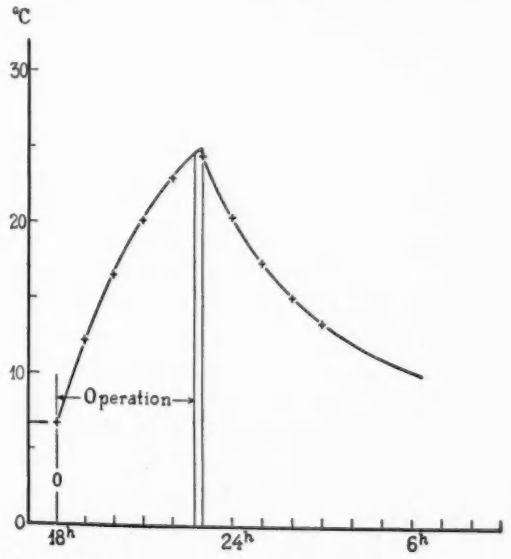

FIG. 10. Surface temperature of a membrane sound transmitter during operation and subsequent pause. Total weight 455 kg, average specific heat 0.115 kcal./kg.

In addition the vibration measurements give clear evidence that the consumption of energy must be rather considerable; not only were the vibrations perceptible with the finger tips on practically the entire inner surface of the tower, but the vibrations of the solid rock were easily measureable with the crystal pick-up at a distance of 50 meters.

These vibration measurements and the calculation of the vibration losses in the preceding section emphasize the importance of a "floating" mounting for the transmitter, leaving it to move freely without transferring appreciable vibrations to the foundation. It would be of interest to remount the transmitter here concerned and repeat the measurement of the acoustical output to determine the fraction of power gained as sound radiation by elimination of the vibration losses.

9. Acoustical Output of a Diaphone

Measurements of the acoustical output were performed in June 1937 at a fog signal plant consisting of a diaphone with $8\frac{1}{2}''$ piston, operated by compressed air delivered from a tank containing 12,300 liters, the compressor capacity being 5000 liters of free air per minute. The diaphone is operated 4 seconds per minute, the manometer pressure then dropping from 2.1 kg/cm^2 to about 1.6 kg/cm^2.

The axis of the cone of the diaphone is hori-

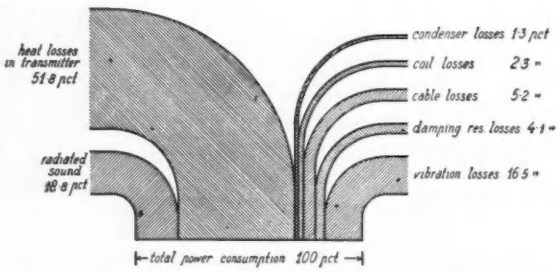
heat losses in transmitter 51·8 pct
radiated sound 18·8 pct
condenser losses 1·3 pct
coil losses 2·3 "
cable losses 5·2 "
damping res. losses 4·1 "
vibration losses 16·5 "
total power consumption 100 pct

FIG. 11. Power balance of a membrane sound sender.

zontal and the opening is situated 7–8 ft. above the cemented ground. The mounting of the instruments for the acoustical measurements were accordingly simple; measurements were performed along two lines at distances of 24 cm and 54 cm, respectively, from the front of the cone. As both series give about the same total output, only the first series is recorded here.

The direction of sound was determined by a Rayleigh disk instrument, while the intensity was determined by a crystal microphone calibrated in the sound field of the membrane sender, as described in Section 4. The figures in the second column of Table II are based upon the measurements performed with the Rayleigh disk instrument, giving the sound pressure in the line of observation of the crystal microphone (19 cm from the opening of the cone). As the e.m.f. delivered by the crystal microphone is proportional to the sound pressure, this will also be the case with the current recorded by the rectifier galvanometer directly connected with the crystal microphone. In fact, the proportionality factor is very nearly constant (=3000 bars per microampere), which indicates a good agreement between the Rayleigh disk measurements and those performed with the crystal microphone with regard to relative intensity distribution in the sound field. Thus we have

$$p=3000i \quad \text{dyne/cm}^2,$$

which again is equivalent to

$$J=0.0217i^2 \quad \text{watts/cm}^2. \qquad (9.1)$$

The intensity curve for the diaphone, calculated by the formula (9.1) from the current delivered by the crystal microphone (at an observation line 24 cm in the front of the cone) is given in Fig. 12*a*. Fig. 12*b* represents the angle between the direction of the sound and the observation line. According to Eq. (5.2) dS/dR is then calculated, which is represented by Fig. 13*a*, and subsequently the flux S_r, represented by the Fig. 13*b*. A section of streamline tubes each carrying 250 watts is given in Fig. 14.

From the last two figures it is seen that the acoustical output of the diaphone in question was *2200 watts*. As the frequency characteristic of the crystal microphone is practically rectilinear below 5000 cycles per second, no correction for the frequency distribution in the sound spectrum of the diaphone is introduced (for sound wave oscillogram see Section 11). We estimate the probable error of the measurement to be about 10 percent.

10. POWER BALANCE OF A DIAPHONE

The energy delivered to the diaphone during the operation period $0\rightarrow t$ will be

$$L=\int_0^t A(dV/dt)dt, \qquad (10.1)$$

the quantity A being the energy delivered per liter air at the actual pressure, and dV/dt being the volume delivered per second, partly from the storage tank and partly from the compressor. The maximum of energy available for mechanical work, acoustical radiation included, will be determined by the equation

$$A=10\frac{k}{k-1}p\left[1-\left(\frac{1}{p}\right)^{k-1/k}\right] \quad \text{kgm}$$

$$=35p[1-p^{-0.286}] \quad \text{kgm}, \qquad (10.2)$$

which represents the work done by 1 liter of air

TABLE II. *Calibration of crystal microphone in the field of membrane sender, observation line 19 cm from cone. Shure microphone, type 85A.*

DISTANCE FROM AXIS cm	SOUND PRESSURE, p dyne/cm²	MICROAMPERE, i	p/i
0	6880	2.26	3040
10	6720	2.22	3030
20	6340	2.15	2950
30	5820	2.01	2890
40	5300	1.84	2880
50	4600	1.54	2990
60	3580	1.18	3030
70	2420	0.80	3030
80	1880	0.62	3030

at the pressure p kg/cm² undergoing ideal adiabatic expansion from the pressure p to atmospheric pressure, 1 kg/cm² (Fig. 15). To perform the (numerical) integration of Eq. (10.1) by (10.2) the pressure p must be expressed as a function of time t, which is conveniently done by the assumption that the drop per second of the tank pressure during the operation of the diaphone is proportional to the manometric pressure $(p-1)$, thus

$$dp = C(p-1)dt. \qquad (10.3)$$

As $p=3.1$ at $t=0$ and $p=2.6$ at $t=4$, the integration of (10.3) delivers

$$C=-0.068 \quad \text{and} \quad p-1=2.1e^{-0.068t}. \qquad (10.4)$$

In Table III are given the values of p and A, respectively, according to Eqs. (10.4) and (10.2),

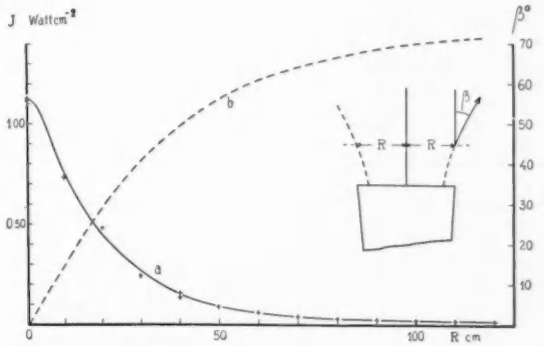

FIG. 12. *a*, Diaphone sound intensity (watts/cm²), measured by crystal microphone 24 cm in front of the cone. *b*, Sound direction according to Rayleigh disk instrument.

corresponding to $t=0.5,\ 1.5,\ 2.5,\ 3.5$ sec. To calculate the values of dV/dt at the same values of t we observe that dV/dt consists of two parts, *viz.* (1) the input from the compressor at the pressure considered, which will be approximately $83/p$ liters per sec., and (2) that delivered from the tank store, $(dV/dt)_a$, which is the chief contribution. To calculate this we notice that the volume of air delivered to the diaphone from the tank due to a given pressure decrease would be equal to the change of volume of the air mass of the tank, if it were submitted to adiabatic expansion within the same pressure range. The tank volume being $V_1=12300$ liters and the initial pressure at the time $t=0$ being $p_1=3.1$

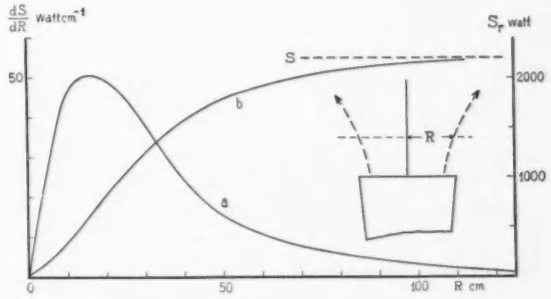

FIG. 13. *a*, dS/dR for a diaphone, according to Eq. (5.2). *b*, Energy flux S_R for a diaphone, according to Eq. (5.3).

kg/cm²,

$$V=\left(\frac{p_1}{p}\right)^{1/k} V_1 = 27620p^{-0.715} \text{ liters.} \qquad (10.5)$$

By differentiation and introduction of (10.3)

$$\left(\frac{dV}{dt}\right)_a = 1343\frac{p-1}{p}p^{-0.715} \text{ liters/sec.} \qquad (10.6)$$

Adding this quantity to the input from the compressor, $83/p$, the value of dV/dt results. Finally, the integration according to (10.1) may be performed numerically, as indicated in Table III. The maximum of energy available for mechanical work is according to Table III 45340 kgm, being delivered in 4 seconds, thus corresponding to an input to the diaphone of *111 kilowatts* during the operation period.

As the acoustical output of the diaphone considered was 2.2 kilowatts according to our measurements, the ratio output/input is 2.2/111

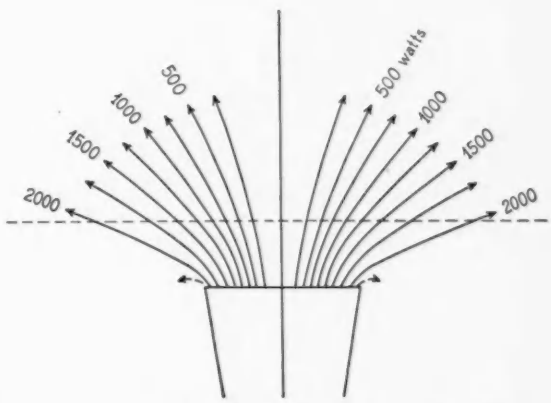

FIG. 14. Streamline tubes for a diaphone, each carrying 250 watts (circular section).

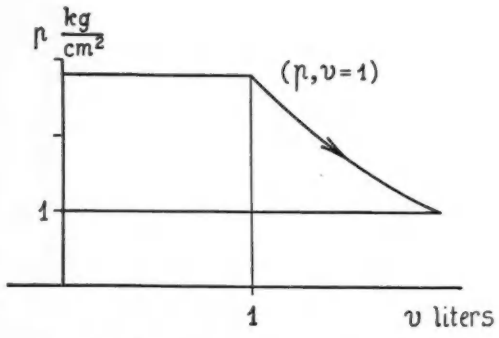

FIG. 15. Ideal cycle between p and 1 kg/cm².

$=0.02$, i.e., 2 percent. If we take into account the fact that the energy is stored during the pause (of 56 sec.), the ratio of output to average input will be $0.02 \cdot 60/4 = 0.30$ i.e., 30 percent.

11. Oscillograms of Sound Wave Form

In most parts of the sound field of the membrane sender (Section 6) the observer will hear a practically pure sound; in the central parts, however, a content of harmonics is easily observable. The intensity curve representing a single cycle is shown by the oscillograms reproduced in Fig. 16. The crystal microphone was here directly connected with the terminals of a cathode-ray oscillograph being synchronized by the fundamental frequency.

In the sound field of the diaphone (Section 10) the observer will hear a penetrating sound giving the impression of a rather complicated sound spectrum, and at the end of each blast

TABLE III.

t	0.5	1.5	2.5	3.5	sec.
p	3.030	2.897	2.772	2.655	kg/cm²
A	28.8	26.6	24.5	22.6	kgm/1
$(dV/dt)_a$	408	412	415	417	1/sec.
$83/p$	28	29	30	31	1/sec.
dV/dt	436	441	445	448	1/sec.
$A\ dV/dt$	12570	11750	10900	10120	kgm/sec.

Total $= \Sigma A\,(dV/dt)\Delta t = 45340$ kgm

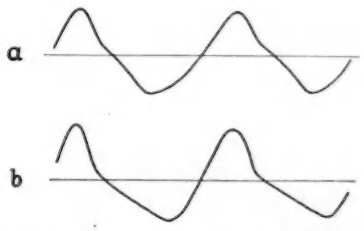

FIG. 16. Sound wave form for a membrane sound sender, cathode-ray oscillograms. a, in the axis, b, 30 cm to the side.

FIG. 17. Sound wave form for a diaphone, cathode-ray oscillogram (in the axis).

the dominating frequency drops down during a "grunt." The complexity of the wave form is demonstrated by the Fig. 17; the synchronization of the cathode-ray oscillograph met with some difficulties, and during the "grunt" synchronization was impossible. Continuous recording would be necessary in that case.

From the shape of the curve of Fig. 17 it is seen that the piston of the diaphone sensibly contributes to sound intensity as a membrane, vibrating at half the frequency of the pressure impulses, thus reducing the resulting intensity when the phases are opposite. It is indeed, astonishing that the contribution to sound radiation of the pressure impulses do not exceed to a much greater degree the membrane action of the piston. The cause is, according to our opinion, that the air currents by passing the piston slits will suffer a violent turbulence, which will effectively counteract the formation of coordinated air oscillation necessary to produce a progressive sound wave. This would also give a reasonable explanation of the low mechanical efficiency of the diaphone and would in addition indicate the way to its improvement.

Some Acoustic Problems of the Automotive Fan

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(Received December 15, 1937)

INTRODUCTION

THE cooling fan, in the past few years, has been on the noise list of most automobile companies. The fan is primarily used to cool the motor, and the specifications are quite rigid in regard to the dimensional and performance characteristics. Noise standards have changed so much that we have known of manufacturers lowering their cooling standards in order to maintain a low noise level of the fan.

About six years ago, fan noise was beginning to take its place among the "nuisance" noises of the automobile. In collaboration with one of the large automobile companies, the asymmetric spacing of fan blades was evolved. Reducing the apparent noise of propellers by spacing was suggested in England by the War Department in the early twenties. There are not many of the modern cars which do not use asymmetric spacing of their fans.

ACOUSTICAL PERFORMANCE

The methods of testing a fan for its cooling capacity are fairly standardized. The instrumentation technique is very accurate and results are easily checked and dependable. However, when we come to measure the effect of the fan's noise spectrum on the spectrum of the whole car, we find no acoustic standards using instrument technique presented by any of the automobile manufacturers.

The method of test used at present consists of installing various designs of fans which will give the required cooling, then road testing the car and listening at different speeds. This procedure is carried out until there is an improvement in the fan's noise level. Then the fan is submitted to the engineers who control the performance quality of the car. They accept it for engineering approval if they agree that the reduction in noise level is sufficient.

Because of the human element involved in these tests, many differences arise. The fallability of the human ear and the variation in opinion of those who judge the quality of the fan complicate the formation of definite conclusions. Therefore, one can see the need of a measurement technique to put fan selection on a standard basis.

BLADE SPACING

Due to the rigid specifications of the fan covering: space occupied; rotational speed; amount of air flow; number of blades; position in car; proximity to surrounding parts, etc., the difficulty in making a quiet fan is obvious.

An automotive noise to be objectionable under the present standards must have one, or all, of the following qualifications:

1. A sudden increase, or decrease, of the noise level as speed is changed. This is known as a period.
2. A relatively high noise level of some single frequency or group of frequencies, perhaps harmonically related.
3. A noise of unusual quality, though the level may be low.

When the automotive test engineer discovers the car with one of these noise characteristics, he tries to decide which part, or parts, are responsible. Those concerned with the design of any parts that give rise to noise must be ready to change or decrease the noise level of their parts involved. The noise level must be either reduced or the spectrum of the noise shifted in regard to the intensity of the components so that it fits better into the complete spectrum of the car.

Our experience has shown that the change of blade spacing is one of the quickest ways to shift the intensity of the fan's acoustic spectrum without altering the physical dimensions.

Figure 1 shows a group of fans used on 1937 and 1938 model automobiles, trucks, and buses. Almost every car today has its own specially designed fan. If it isn't cooling which dictates the design, it is noise or installation dimensions.

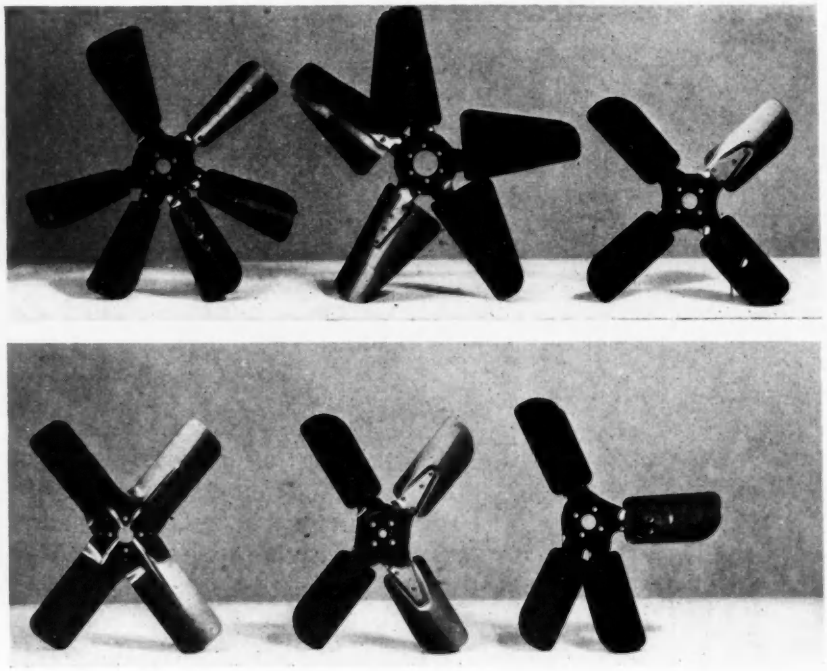

FIG. 1. Group of fans now in production for 1938 cars, trucks, and buses.

Figure 2 shows three fans. These are identical except for their spacing. The airflow and horsepower are also the same. The 70 degree spaced fan is used on a production 8-cylinder car and the radically spaced type is used on the same motor but with a different chassis and body. The 90 degree fan is made for a truck engine.

Figure 3 shows the spectrum of the principal frequencies. These spectra were obtained in the laboratory with a General Radio wave analyzer. The microphone was placed on a line with the axis of rotation, $7\frac{3}{4}$ inches from the tip of the fan and slightly to the front or intake side. The variation in loudness of the harmonics of the three fans is pronounced. It will be noted that the measured frequencies are, 40, 80, 120, 160, 240, and 320 cycles, respectively. This corresponds to a rotational speed of 2400. Of course, there are other harmonics, but these shown measured the loudest. These readings were not taken in conjunction with a weighing network, which would alter the spectrum somewhat.

The radically spaced fan shown in Figs. 2 and 4 was a design developed to eliminate a fre-

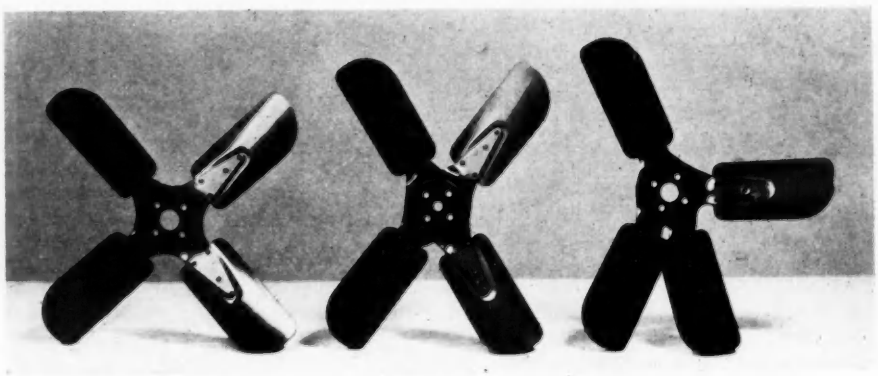

FIG. 2. Types of commercial fans with various spacings.

quency band that was predominant in the car's spectrum at about 50 m.p.h. This fan replaced the 70 degree spaced fan of Fig. 2 whose emitted frequencies were not masked at that speed by other car noises or which had a noise pattern in the body increased due to resonance at those particular frequencies caused by the body cavity. The new design was satisfactory and the spectrum, Fig. 3, shows the decided change in the frequency level when the 70 degree fan is compared to the radical type. The rotational fre-

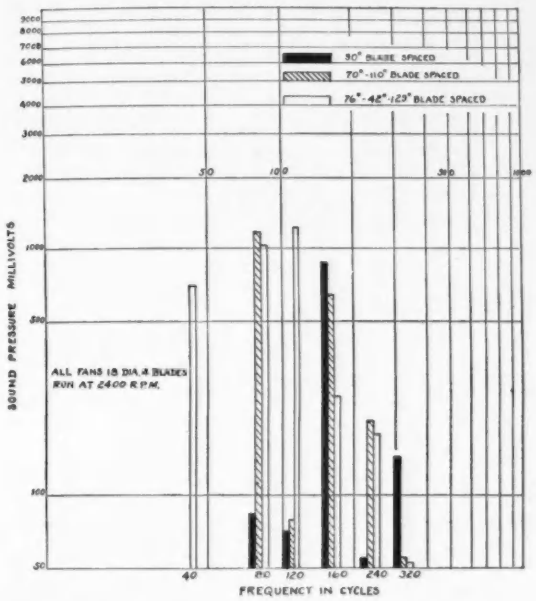
90° BLADE SPACED
70°-110° BLADE SPACED
76°-48°-129° BLADE SPACED
ALL FANS 18 DIA 4 BLADES RUN AT 2400 R.P.M.
SOUND PRESSURE MILLIVOLTS
FREQUENCY IN CYCLES

FIG. 3. Noise spectrums of fans illustrated by Fig. 2.

quency of 40 cycles appears due to the spacing. This fan is now a production part, and in spite of its nonsymmetry, its balance is held to $\frac{1}{8}$ inch ounce.

Figure 5 shows an 18 inch diameter fan made so that it is adjustable in respect to angular spacing. This fan was run at 2400 r.p.m. and the harmonics measured every two or three degrees of spacing. The type of angular change is the same as in placing two, two-blade fans together and then rotating them about their centers. Commercially, this allows a variation of from ninety to about fifty degrees between the blades.

Figure 6 is a plot of sound pressure against this angular variation and shows the rapid change in sound pressure of the different har-

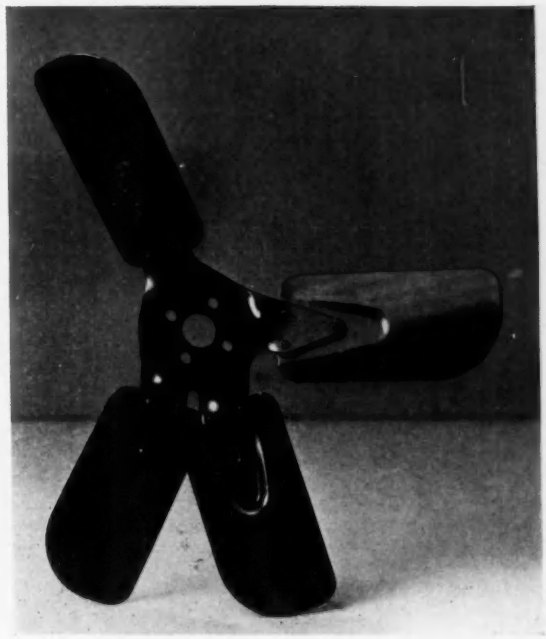

FIG. 4. Radically spaced type of fan developed for a special noise condition in a 1938 model.

monics. This large change in intensity of the harmonics shows why there is such a difference of noise characteristics and level when this type of fan is tested on a car.

Figure 7 shows an 18 inch diameter, 4-bladed, truck fan.

Figure 8 gives the change in intensity of the fan which is spaced at 90 degrees. The r.p.m. was varied and the intensity of the loudest component was measured. In this case it is the number of blades multiplied by the revolutions per second. There is a large change of intensity with r.p.m. or tip speed. The reduction in decibels in relation to the r.p.m. or tip speed as shown in Fig. 7 follows fairly well those results obtained by A. H. Davis in England on aircraft propellers.[1] Davis found that for a given propeller, a reduction in tip speed of one hundred feet per second gave an average reduction of 10–15 decibels. This relation seemed to hold for different tip speeds and noise levels. We stress low rotational speed for any design we contemplate and find that slow running fans average the quietest.

Figure 9 shows a fan used on an expensive

[1] J. Roy. Aero. Soc. **35**, 676 (1931).

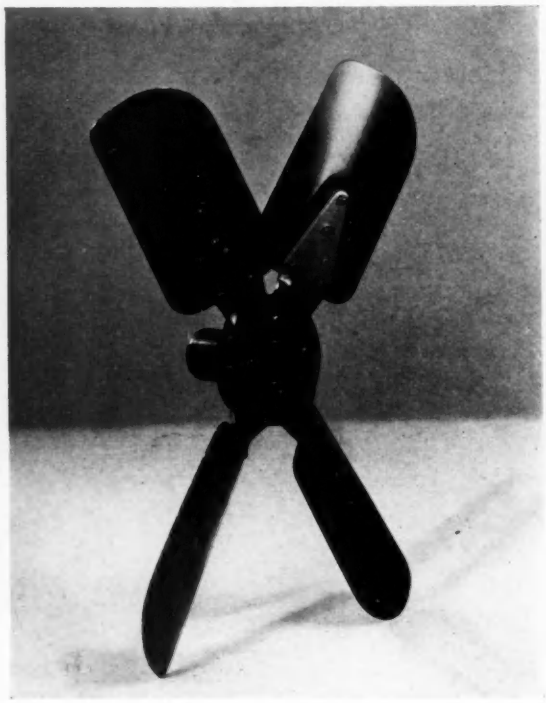

FIG. 5. Adjustable model for determining effect of spacing on a fan's noise spectrum.

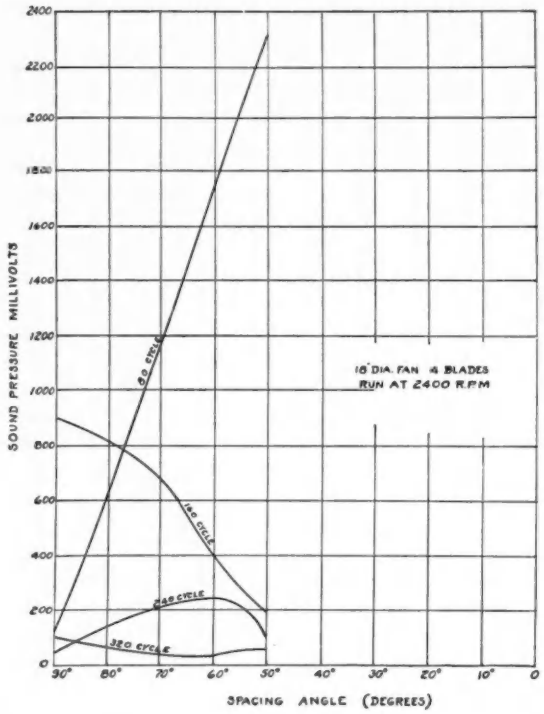

FIG. 6. Result of changing angle between the blades of fan in Fig. 5.

FIG. 7. 90-degree spaced fan used on a truck engine.

1938 model, and Fig. 10 gives the frequency analysis. It is interesting to note here that although the fan is seven bladed, two of the blades are so close together that they act as one blade and the analysis is that of a six-bladed fan with the harmonic loudness of different values due to the irregular spacing.

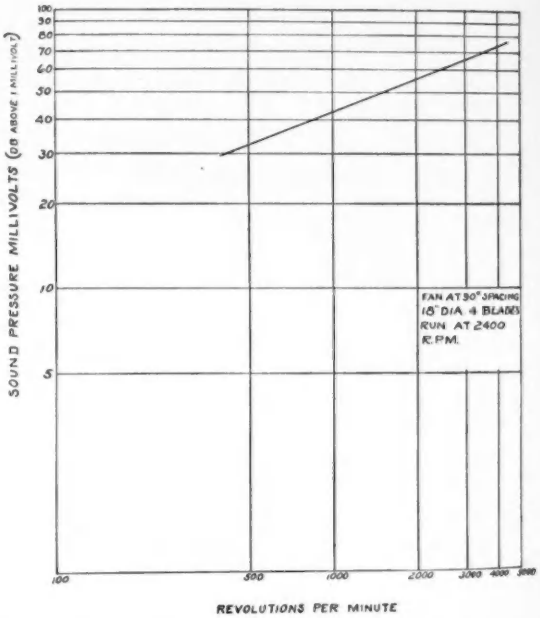

FIG. 8. Change of noise level with r.p.m. of fan illustrated by Fig. 7.

With the large diameter multiple-blade fans, which are often used on the larger cars to give greater airflow, there is apt to be predominance of the higher frequencies with possible difficulty in changing their loudness levels. This is also true when smaller diameter fans are run too fast in relation to crankshaft speed. The frequencies are so high that though there is a spectrum shift due to any spacing we apply, the change to the lower frequencies is not enough to be masked and often the increased intensity of slightly lower frequencies accompanies an apparent loudness which makes the fan worse.

The present automobiles at high speeds have many low frequency sounds, which come from wind noise, road conditions, motor vibrations, etc., and by running the fan as slowly as possible,

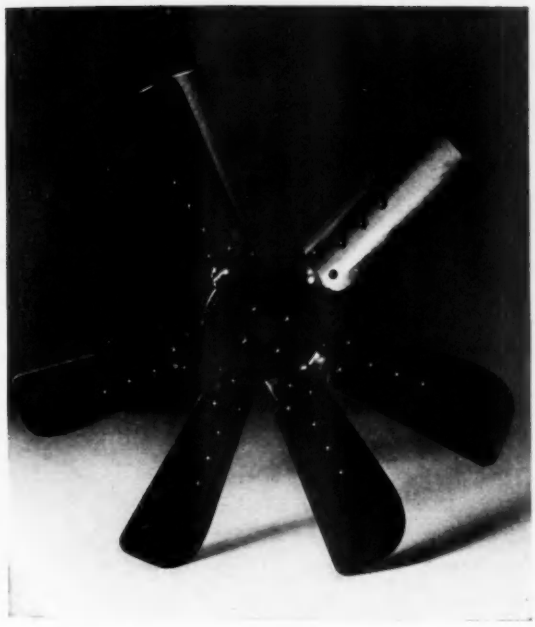

FIG. 9. Fan used on expensive 1938 model car.

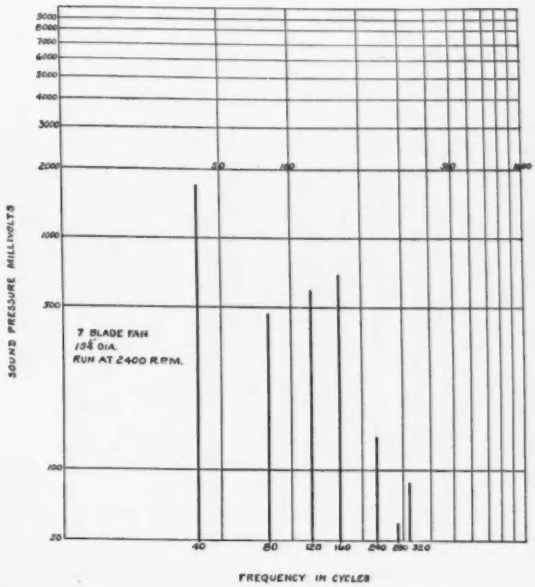

FIG. 10. Noise spectrum of fan shown in Fig. 9.

and with proper spacing, one can often mask the low frequencies of the fan by these noises. This necessitates larger and more powerful fans which will give the needed airflow. Even then the fact that fan noise has a peculiar quality of its own makes it difficult to mask behind other car noises.

With progress in car design, the levels of these various noises will be gradually reduced and the fan noise will then be increasingly predominant if proper design is neglected. Unless a cheaper method of cooling a car is devised, more attention must be paid to the installation of the fan if the proper acoustic performance is desired. When methods of tests are found to give the allowable noise spectrum for a fan, in relation to the total spectrum of the car, we expect to have sufficient data to furnish the required analysis.

The Vibrations of Symmetrical Plates and Membranes

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(Received March 9, 1938)

The mathematical theory of vibrating membranes or plates, which is usually applied to squares and rectangles, may also be used for triangles provided the equations are plotted on axes which are not rectangular. These triangles are then fitted together to produce many sided symmetrical plates. Thus the Chladni patterns are found mathematically for polygonal membranes or plates. If one angle of the triangle is made very small, the plates are practically circular so that these equations also give the patterns on circular plates (approximately) for special cases. The triode valve oscillator which drives the plates is calibrated accurately so that the frequency of the plates is found at the time the patterns are formed. A table gives a comparison between the observed and calculated frequencies.

IN 1882 Elsas[1] made an extensive research upon vibrating membranes. He had at his disposal thirteen small tuning forks and four larger ones. Using the overtones of some of these forks, he obtained in all forty different tones. He found great difficulty in vibrating large membranes, so he confined his experiments to round plates 15 centimeters in diameter and square plates 15 centimeters on each side. A thread attached to the fork and the center of the membrane acted as the coupling between the two, the membrane being forced into oscillation by the vibrating fork.

In the vacuum tube oscillator,[2] the audible notes of a triode valve are sent through a power amplifier which delivers 12 watts at the loudspeaker. The amplified note is so loud that the membrane can be set in vibration by the sound waves alone. Sometimes, however, one end of a small copper rod about three inches long was cemented to the center of the diaphragm of the loudspeaker, the other end being pressed against the membrane. In this way membranes ten inches across may be vibrated. This size was chosen because it corresponds with the dimensions of the square Chladni plates used in previous experiments. It is extremely difficult to obtain nodal lines upon any of the large membranes. This is probably due to the fact that as the amplitude of the vibrating membrane increases, the tension T is no longer a constant and the sand figures become distorted. Elsas always vibrated his membranes from the center so that all his figures were symmetrical about that point. The copper rod was applied to various points on the membranes and some figures obtained were not symmetrical about the center. A few of these are shown in Fig. 1. The membrane in the first four photographs was circular but was pasted upon a hexagonal frame. The frame is shown in two of the pictures. Only one of the figures on the square membranes shows symmetry about the center although the others are symmetrical about median lines or diagonals.

The mathematical theory of vibrating membranes is well known although it will be shown here that it is not complete because no account has been taken of the fact that each segment of a square or circle may also vibrate as a unit. Hence, many new figures which have formerly escaped notice may be developed from the mathematical equations. The equation for the nodal lines of a square membrane (side a) with fixed edges takes the form

$$w=A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{a}+B\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{a}=0. \quad (1)$$

If the values $m=5$, $n=2$, $A=10$, $B=1$ are substituted in this equation, the nodal lines appear as shown in Fig. 2a. If this figure is cut into two equal parts by a median vertical line and the right half inverted, a new figure (Fig. 2b)

[1] Elsas, Nova Acta der Ksl. Leop. Carol. Deutschen Akademie, Vol. 45, No. 1. Halle 1882.
[2] R. C. Colwell, Phil. Mag. **12**, 320 (1931).

is found for the same conditions. This process may be continued indefinitely provided the figure meets the requirement that the left-hand half of each figure when rotated 180° will fit exactly over the other half.

A slight extension of Eq. (1) gives an equation for a rectangular membrane in the form

$$w=A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}$$

$$+B\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{b}=0. \quad (2)$$

The nodal lines of the rectangle $m=5$, $n=2$, $A=8$, $B=1$ are shown in Fig. 3a for $a=2b$. The rectangle may be cut into two squares and these fitted together in such a way as to produce Fig. 3b. The rectangles may then be fitted together to form squares. If $a=3b$, $4b$, etc. it is possible to build up many other nodal patterns for a square membrane. These smaller units from which the complete membranes are formed will be called mosaics. They can be fitted together only when they satisfy the proper boundary conditions and all have the same rate of vibration as given by m and n of Eq. (2).

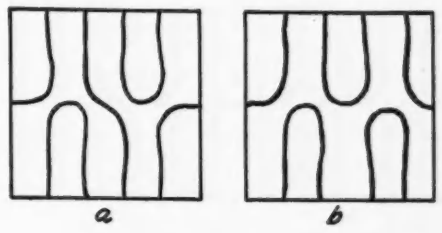

FIG. 2.

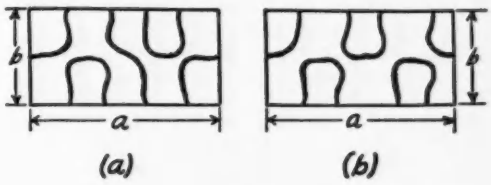

FIG. 3.

If a triangle is chosen as the mosaic, it is possible to build up the vibrational systems of symmetrical membranes which are in the form of pentagons, hexagons, etc. It will be noticed, however, that these are special symmetrical solutions and do not apply generally. In fact the general solution for a many-sided symmetrical membrane is one of very great difficulty. If the coordinate angles are taken at 60° (Fig. 4), the membrane *OBCD* is a rhombus with an acute angle 60°. The equation for such a mem-

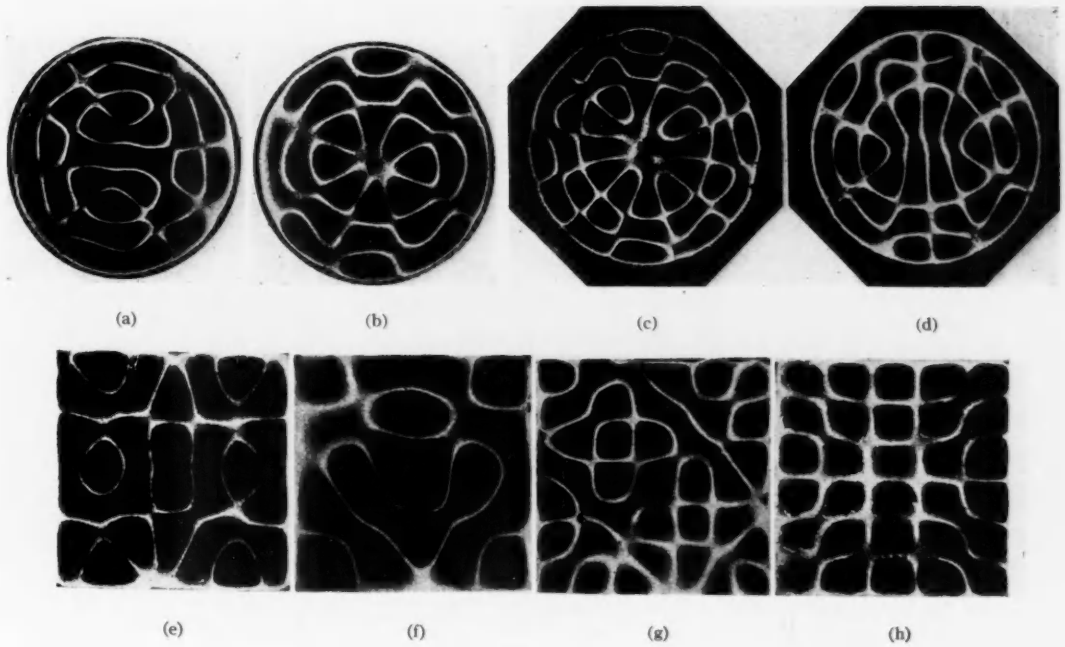

FIG. 1.

brane is

$$w=A\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{a}+B\sin\frac{n\pi x}{a}\sin\frac{m\pi y}{a}=0 \quad (3)$$

but the coordinates are no longer rectangular.

Nodal patterns (Fig. 5) are plotted from this equation and those chosen for mosaics which have the diagonal *BD* an antinodal line. This

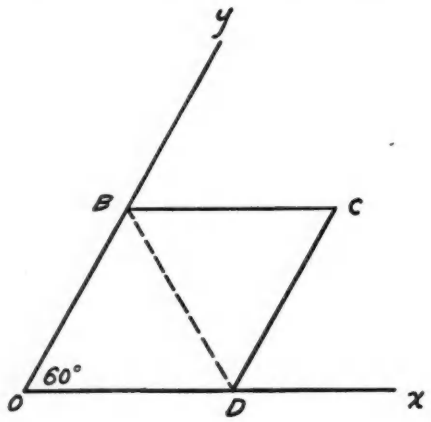

FIG. 4.

insures that the sides *OB* and *OD* are fixed, while the diagonal *BD* is free. Hence, the completed figure represents a hexagon with diameters as nodal lines and the edges lines of maximum vibration. Such a membrane cannot be realized physically since in a membrane the edges must be fixed; but the equations are immediately applicable to plates in which the edges are free. In Fig. 5 this mosaic is shown for values $A=B=5$, $m=3$, $n=5$. Beside it is the hexagon made up of six similar triangular mosaics. The diameters of the hexagon are necessarily nodal lines. For a membrane in which the diameters are not to be nodal; it is necessary to choose a mosaic in which *OB* and *OD* (Fig. 3) are free edges. This condition gives an equation

$$w=A\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{a}+B\cos\frac{n\pi x}{a}\cos\frac{m\pi y}{a}=0. \quad (4)$$

If many-sided figures are required it is necessary to make the angle *BOD* much smaller (Fig. 6). In particular if the angle *BOD* is very small, the periphery of the resulting figure approaches that of a circle. Hence, by the proper combination of these mosaics it is possible to get many figures which should appear upon a circular membrane (Fig. 7). The general solution for the whole circular membranes permits only circles and diameters but the fact that a membrane may vibrate in segments introduces many other types of symmetrical figures which have not been explained before.

To produce the figures experimentally a vacuum tube type beat frequency oscillator was designed. The circuit diagram (Fig. 8) shows two type 6C5 triodes. One of these was operated at a fixed frequency oscillation (175 kilocycles), while the other acted as a variable frequency oscillator. The output of these two high frequency oscillators was injected into a special mixer tube (6L7) on two separate shielded grids. Each oscillator was well shielded from the other one so as to avoid interaction, when the difference between the two frequencies became small.

The plate current of the mixer tube contained the original two frequencies plus the sum and difference frequencies and some harmonics of each. By means of a low pass filter all frequencies above the audiospectrum were eliminated from the output. At the higher audiofrequencies the filter therefore, operated to eliminate the high harmonics as well as the two beating frequencies. Due to this fact the output wave form was

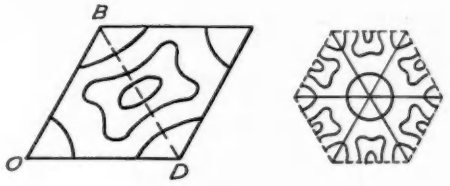

FIG. 5.

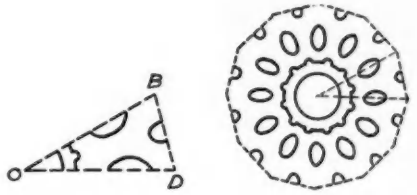

FIG. 6.

quite excellent over the frequency range between 250 and 12,000 cycles per second. Between 40 and 250 cycles per second the wave form became progressively more peaked as the frequency was decreased.

The output power of this device was not great enough to drive a speaker or a vibrating rod with sufficient energy to vibrate large plates and membranes, hence, a standard high quality amplifier capable of stepping up the power to 25 watts was used. The output of this device could be employed either to excite a standard dynamic speaker unit to which was attached a driving rod or a magnetostriction driving unit. For stabilization of the amplifier output circuit, a resistor was connected to a third winding on the output impedance matching transformer so as to dissipate from one-fourth to one-half of the total output power of the amplifier.

For operation on the low and medium audiofrequencies the dynamic speaker unit gave much the better results; but when it was desired to vibrate plates at high audiofrequencies it was found that the magnetostriction driver was more satisfactory. This device consisted of a pair of layer wound solenoids through which a steel rod was passed. The magnetostrictive effect caused the free ends of the center clamped rod to vibrate in unison with the applied audiofrequency.

The chief difficulty arising in the use of the magnetostrictive oscillator was due to the resonance effects encountered. With a certain length rod resonant at a given frequency, it was found that when the exciting frequency approached the resonant frequency the vibrations were much stronger than usual. With this resonance effect in operation the harmonics of the driving frequency were sometimes accentuated enough to require a check on the actual vibration frequency. This was obtained by the use of a

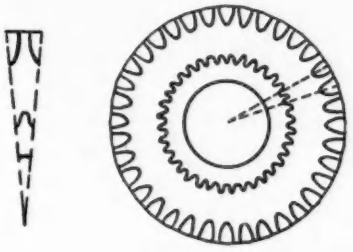

FIG. 7.

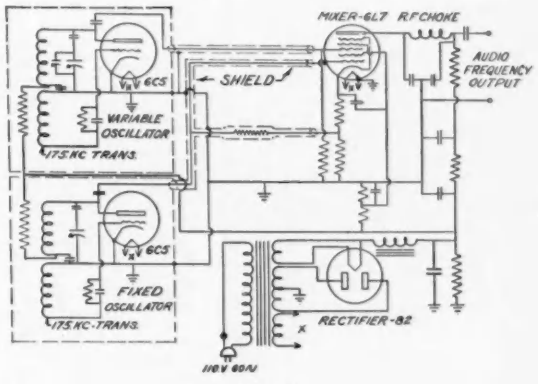

FIG. 8.

Rochelle salt crystal vibration pick-up device with a second amplifier and a cathode-ray oscilloscope. By applying the picked up vibration frequency to one set of oscilloscope plates and a portion of the original driving signal to the other plates, a pattern of Lissajou's figures was obtained from which the harmonic number of the vibration could be deduced.

The beat frequency oscillator (Fig. 8) was calibrated against standard frequency sources, so that it was an easy matter to find the vibration frequency of the plates or membranes by reading the oscillator dial.

The general solution for the vibration of square Chladni plates gives both trigonometric and hyperbolic functions; but at fairly high frequencies the hyperbolic functions disappear so that the solution for the plate is similar to the solution for a membrane with free edges and takes the form given in Eq. (4). To produce the figures experimentally the plates are balanced upon the end of the magnetostrictive rod and the resulting nodes are shown by the position of the sand. A few examples are given in Fig. 9. It is possible to calculate the frequency of vibration of these plates from the formula

$$\text{Frequency} = \frac{\pi}{2}\left(\frac{D}{\text{mass per unit area}} \times \left(\frac{m^2}{a^2}+\frac{n^2}{b^2}\right)\right)^{\frac{1}{2}}, \quad (5)$$

in which $$D = \frac{Eh^3}{12(1-\sigma^2)}.$$

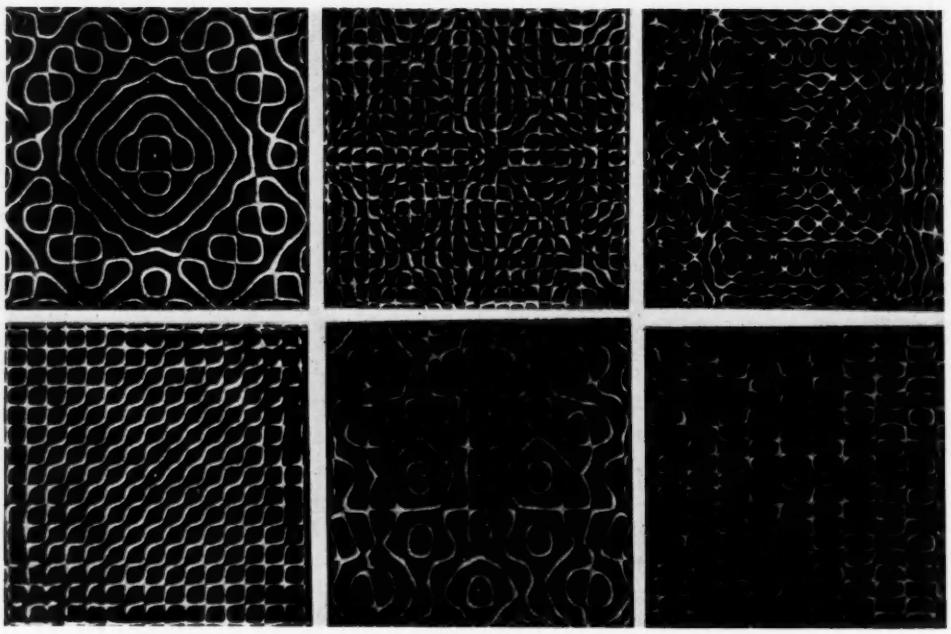

FIG. 9.

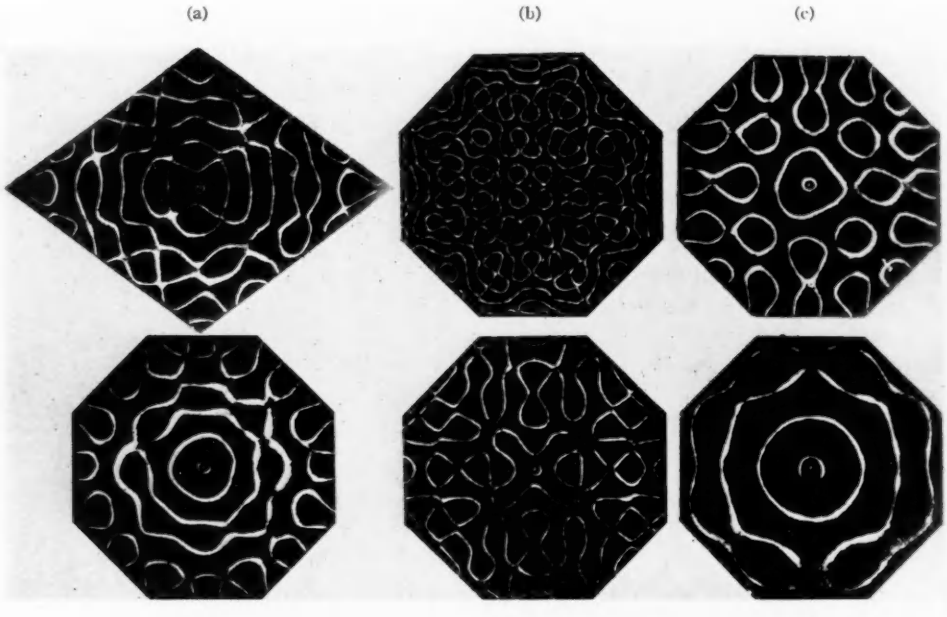

FIG. 10.

σ is Poisson's ratio; E is Young's modulus; h=the thickness of the plate, $a=b$=the length of the side of the square plate. The computed frequencies are compared with the observed frequencies in Table I. Since the magnetostrictive rod occasionally produced an harmonic stronger than the fundamental, it is necessary to use the proper multiplying factor. With the exception of the last one (Fig. 9f), the calculated and observed values are in fair agreement.

All the conclusions that have been made for membranes can be applied directly to metal plates at high frequencies; hence, many of the figures can be calculated mathematically by dividing the plate into triangular parts and finding the nodal lines on each triangle. Also triangles may be formed from many-sided figures as shown in Fig. 10 and these triangles may be put together to form many-sided figures.

TABLE I. *The frequencies of the plates in Fig. 9.*

PLATE	m	n	FUNDAMENTAL FREQUENCY	MULTIPLYING FACTOR	CALCULATED FREQUENCY	OBSERVED FREQUENCY
Fig. 9a	12	12	4341	1	4341	4880
b	22	18	2787	4	11148	10450
c	28	28	2734	2	5468	5460
d	16	16	2465	2	4930	5360
e	16	10	4827	1	4827	4900
f	20	10	3406	?	3406	5370

Acoustical Society News

(The Editor will appreciate receiving items of news about scientific matters, honors or prizes given to members of the Society, etc. Address such notes to F. R. Watson, University of Illinois, Urbana, Illinois.)

THE Washington meeting of the Society was held May 2–3, with a program of 27 papers. Most of the sessions were held at the National Bureau of Standards, but on Monday afternoon, a trip was made to the U. S. Department of Justice where interesting descriptions were given of the work of the "G-men." The dinner Monday evening at the Wardman Park Hotel was attended by some 70 members, and this was followed by the papers as announced in the program. The success of the meeting was due to the efficient management of Mr. V. L. Chrisler of the National Bureau of Standards.

Officers elected were: F. A. Saunders, President; F. A. Firestone, Vice President; Wallace Waterfall, Secretary; G. T. Stanton, Treasurer; F. V. Hunt and Don Lewis, Councilors to succeed H. C. Hayes and A. T. Jones whose terms of office expired.

Future meetings of the Society will be held at Harvard University, November 18–19, 1938; New York City, about May 1, 1939, and at State University of Iowa, Iowa City, in the fall of 1939. It is planned to make the New York meeting a celebration of the tenth anniversary of the Society.

A Cumulative Index of all the publications in the journal during the ten-year period will be published in the journal in 1939. This convenient index is being prepared under the direction of Professor Firestone, in cooperation with the editorial staff of the American Institute of Physics.

Book Reviews

The New Heating, Ventilating and Air Conditioning Guide. Published by the American Society of Heating and Ventilating Engineers, 51 Madison Ave., New York City, 840 pages, 1938, cost $5.00.

The Heating, Ventilating, Air Conditioning Guide 1938 is the 16th edition of the official reference book compiled by the American Society of Heating and Ventilating Engineers. It is published for engineers, architects, contractors, schools and colleges, purchasing agents, manufacturers, public utilities and others engaged in the field of heating, ventilating and air conditioning.

The 1938 edition contains 840 pages of technical reference data included in 45 chapters covering material on design and specifications of heating, ventilating and air conditioning systems. Important new material which has been added on the cooling phases of air conditioning practice, includes extensive revisions of chapters on refrigerants and air drying agents, cooling load determinations and design of central systems for cooling and dehumidifying. Noteworthy is the fact that a chapter on air conditioning in the treatment of disease appears for the first time. A new feature is a visual chapter index.

In addition to the technical material, over 300 pages of manufacturers' catalog data are included, as well as an index to modern equipment listing 300 items, cross-indexed as to product and manufacturer. A complete listing of society members, including their professional or business connections forms a separate section.

Approximately 100 different experts assisted the Guide Publication Committee of the society in compiling this year's volume. The personnel of the committee is Albert Buenger, chairman, S. H. Downs, S. S. Sanford, C. H. B. Hotchkiss and W. H. Severns, and E. N. McDonnell, advisory.

Science and Music. SIR JAMES JEANS. Pp. 258+x, Figs. 64, $14\frac{1}{2}\times 22$ cm. The University Press, Cambridge, and the Macmillan Company, New York, 1937. Price $2.75.

A book on the scientific basis of music by the distinguished Sir James Jeans is an event of unusual interest both to musicians and to acousticians. Written "to convey precise information in a simple non-technical way," the book is an excellent account of the important discoveries in the science of music, including especially a popular treatise of the monumental but too inaccessible *Tonempfindungen* of Helmholtz (1862), the wealth of material added to this book by its English translator, A. J. Ellis, and also the more significant contributions to musical acoustics since then.

The reviewer hopes and believes that Jeans' book is a herald of a renaissance in the scientific study of music. From Pythagorus to D'Alembert it was the vogue for the great scholars of natural philosophy to pursue the scientific foundations of music. Since then, physicists have been attracted to other fields, such as mechanics, light, electricity and electromagnetism, the molecule, the atom, the electron, and the nucleus. But the recent developments in electro-acoustics and in the nature of hearing presage a new era for music and musical instruments. Physicists,

physiologists, psychologists, and engineers can contribute much to this new era of music. Jeans, in this recent book, has set a good example. To the few who are concerned with these new developments, as well as to the many who are interested in the musical scales, instruments and harmonies of today, the Jeans' book will be both instructive and stimulating.

After a brief discussion of the evolution of hearing and the nature of musical sounds, the author presents, among many other things, a brief account of standards of musical pitch (pp. 21–24), which indicate that "standard" *a* varied from 393 cycles in Silbermann's great organ in Strassburg Cathedral (1713) to 567 for the "church pitch" of Northern Germany (1619); a brilliant nonmathematical treatment of the vibration of strings (pp. 61–106), including the application of Fourier series (graphical) to a determination of the frequencies and amplitudes of the *allowed* vibrations of a plucked string, and including also an "aside" on the Hammond electric organ; a modern explanation of the action of flue organ pipes (130–142); an excellent discussion, with valuable original comments, on musical scales (160–190), in which he concludes that "music will have to go very far before finding a better scale" than the present 12-note one which "has its roots embedded very deeply in the unalterable properties of numbers," . . . "yet if ever music becomes independent of the human hand, may not the race then elect to use a continuous scale in which every interval can be made perfect;" and two concluding chapters on the acoustics of concert rooms and the physiology of hearing, chapters which will prove of value to musicians but which contain very little not already recorded in the *Journal of the Acoustical Society.*

Although not inerrant, a shortcoming which readily can be overlooked when it is recognized that in this day of intense specialization it is a far cry from astronomy to acoustics, this book by Jeans will give musicians a good insight into the physical properties of music and also will go a long way to encourage scientists and engineers to devote themselves to the new and important problems in musical acoustics.

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University of California at Los Angeles

A Correction

In the April, 1938, Acoustical Journal in the article: "Speaker Impedance in a Closed Pipe," the following correction is reported by the author:—on page 310, third line above Table I, the statement "constant voice coil current" should read "constant F_0."

Current Publications on Acoustics

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Reviews of Contemporary Papers

MEMBERS who have occasion to translate important papers from foreign journals will be doing a real service to those members who do not read the foreign languages easily, if they will prepare a review of such articles for publication in this section.

These reviews should be of the nature of a lengthy abstract (500 to 1500 words) rather than a critique or appraisal, and should attempt to set forth in this limited space as much of the original author's contribution as possible. One or two figures may be included if desired.

To avoid duplication of effort it is desirable to arrange with the office of this department before writing a review.

Transitions of Sound in the Organ. F. TRENDELENBURG, E. THIENHAUS, E. FRANZ, Akustische Zeits. **3**, 7–20 (1938). —In a previous publication[1] the authors described the beginning of the vibrations in individual organ pipes. These are some of the results they obtained: Reed pipes have very short building up time, flue pipes build up slowly. The principal 8′, for instance, in the Eosander Chapel in Berlin reached 0.8 of its final amplitude only after 0.6 sec. In some cases the tone started with a frequency quite different from the fundamental or its harmonics, which disappeared again in the steady state. (In case of the *Lieblich Gedackt* this so called "antecedent" was particularly pronounced and of a frequency about $5\frac{1}{2}$ the fundamental.) The next step was to investigate the transient phenomena, whilst tone sequences are played. To avoid transport of the extensive measuring apparatus, the various passages played on the organ have been recorded on cinema film. They were played off in the laboratory afterwards, the electric output being passed through a six-range octave filter analyzer.[2] The six filters were connected to the loops of an oscillograph, the amplitudes in the six frequency ranges being thus registered simultaneously. Fig. 1 shows a record for the *Lieblich Gedackt*. The above mentioned "antecedent" is quite well noticeable in the range 1200–2400 c/sec. and also subjectively seems to be of great importance for the peculiar character of these stops. This transient oscillation sharply marks the beginning of the new tone. The record further shows that successive tones are beating with each other. This is not surprising as the vibrations only gradually die away. A low beat frequency causes the impression of rattling or rolling as can be noticed when playing a scale. Higher beat frequencies make the tone sound harsh. Fig. 2 gives a record for the principal 8′. It is quite interesting to note the effect of the large building up time of these pipes. The fundamental hardly appears at the end of the first tone. The second tone in the sequence is building up already quicker whilst only at the end of the third, the fundamental reaches something like the steady state value. This seems to be a certain kind of pulling in effect, always present if the frequencies are not very different. The previously played tones which are gradually dying away apparently accelerate the building up of the vortices at the mouth of the pipes, the steady state therefore being attained in a relatively very short time.

The most important factor for the brightness and quality of an organ seems to be the intonation of the pipes. This is shown up by these old baroque organs which sound brilliant, even if the all-around acoustics are poor. The reverberation, the intonation of the pipes, the various pipes in the stops, the tones played at the same time and those previously played, the construction of the tune, etc.

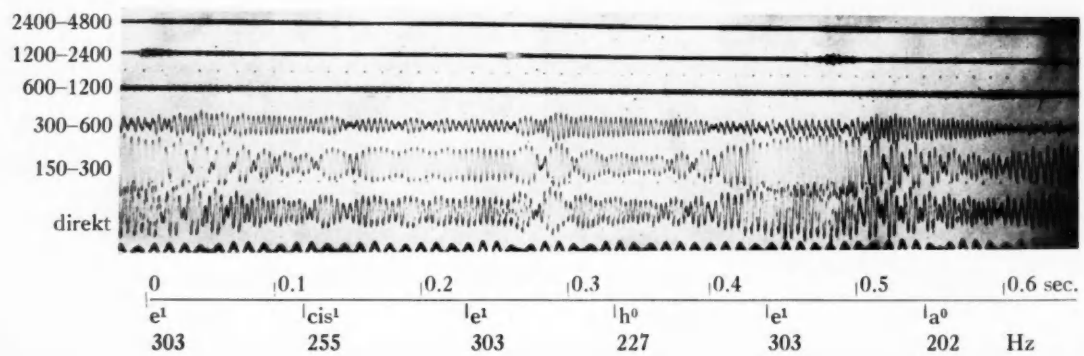

FIG. 1.

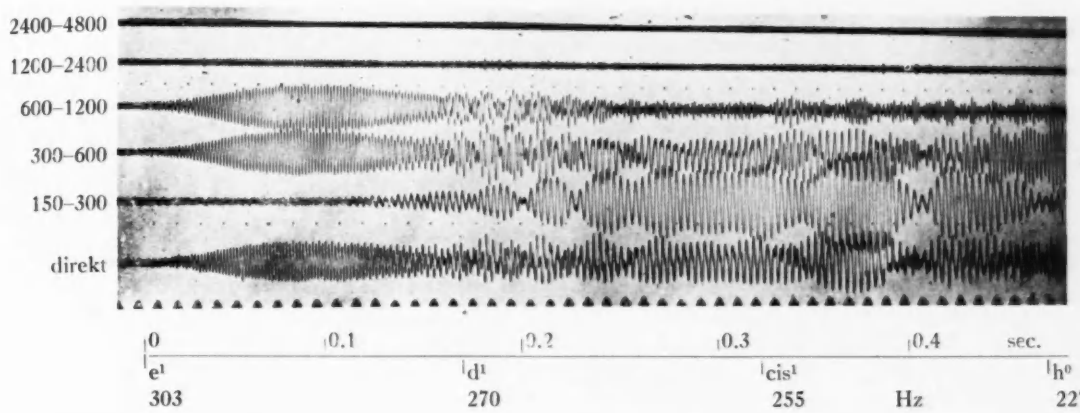

FIG. 2.

greatly influence the transients. Considering that not only the fundamentals but also the overtones are interacting with one another, one can realize the great number of possibilities in organ play. These seem to be the reasons for the great brightness and liveliness of valuable organs.

The beginning of the vibrations in organ pipes plays an important part in the intelligibility of the music, as old experiences testify. The Cembalo for instance, in spite of its small loudness, may lead the rhythm of a relatively large string orchestra, and this only because of its sharp beat-like onset. It can be said that it is desirable, in general, to accentuate the beginning of the vibrations as this stresses the structure of the music and adds to its liveliness. A sharp beat-like onset is obtained by increasing the relative high frequency amplitudes at the beginning of the vibrations. This can be done by means of a proper intonation, or by grouping suitable pipes into the stops. High pitched pipes have short transients, they practically sound at the instant the key is depressed, whilst the vibrations in large pipes build up slowly. A combination of two or more such pipes then leads to the desired effect. There seems little doubt, that the old organ makers observed the mentioned points. They all seemed to have agreed also in their technical measures. They aimed to get small pipes with very short transients and large pipes with relatively long transients, using a low blast pressure, and large scales. A precarious consideration of such and similar details then leads to those wonderful baroque organs which even surprise now-a-days by their brilliance. In contrast to this is the lack of success of the organs constructed about 1900. Organ makers deviated from the old traditions, worked with greater blast pressures, small scales, and avoided the higher partials in the mixed stops and high partial stops.

E. J. SKUDRZYK

[1] Abstract, J. Acous. Soc. Am. **8**, 264 (1937).
[2] F. Trendelenburg, J. Acous. Soc. Am. **7**, 142 (1935).

About the Origin of the Distance Impression in Hearing. G. v. BÉKÉSY, Akustische Zeits. **3**, 21–31 (1938).—One of the acute problems in modern sound transmission is the reproduction of the spatial character of sounds. Direction as well as distance impression attribute to the liveliness of the performance and are appreciated by the individual observer. In one channel transmission or monaural audition the sense of direction is lost, but the illusion of distance still retains its importance. Various factors have been found that influence the "acoustical depth," but there are no publications stating the actual physical magnitudes which determine them.

It is known from broadcasting that one of the factors controlling the acoustic perspective is the ratio of direct to reverberant sound. Since the reverberant sound may be attributed to image sources behind the walls it seems probable that the larger the relative portion of reverberation the greater will be the apparent distance of the sound source from the microphone. This is investigated as follows: A damped room containing the source and a microphone is separated from a reverberation chamber by means of a partition wall having an insulation of about 20 db. Owing to the transmission of this wall a reverberant sound field is established in the reverberation chamber, which contains the second microphone. The two microphone currents are amplified to equal loudness and are passed through a telephone. By means of a potentiometer arrangement their relative portions can be varied, maintaining the loudness in the telephone at its constant value. As to be expected, an increase in the relative portion of reverberant sound increases the apparent distance of the sound source from the microphone. In addition, this causes the impression as if the linear dimensions of the source would have been increased at the same time.

However, this is an illustration rather than the cause for the distance effect, as can be concluded from the fact, that the sensation of distance is even more pronounced in the open air. There are a number factors which, as can be shown experimentally, have no effect on the distance impression, such as the ponderomotive forces between sound source and tympanic membrane, the reflections between the human head and the sound source, and the distortion of the sound field in the neighborhood of the human head.

To find those factors which may produce the distance impression, let us consider the simple case of the pulsating sphere. Its velocity potential is given by

$$\varphi = (1/4\pi r) f(t - r/c). \qquad (1)$$

From this we obtain for the pressure

$$p=\rho(\partial\varphi/\partial t)=(\rho/4\pi r)f'(t-r/c) \tag{2}$$

and the particle velocity

$$u=-\frac{\partial\varphi}{\partial r}=\frac{1}{4\pi}\left[\frac{1}{cr}f'(t-r/c)+\frac{1}{r^2}f(t-r/c)\right]. \tag{3}$$

$f(t)$ represents the volume flow, for

$$\lim_{r\to 0}[4\pi r^2 u]=f(t). \tag{4}$$

The only magnitude which depends on the distance, the decrease of loudness being compensated, is the particle velocity and its phase with regard to the pressure. If the receiver is a pressure actuated microphone there will therefore be no distance effect. If on the other hand the receiver picks up a certain proportion of the velocity component, distance hearing will be expected. To prove this, two telephone earpieces were used as microphones. One fitted with a heavy membrane (0.8 mm thick) acted as a pressure microphone, the other equipped with a very thin membrane permitted a certain amount of velocity reception. With the velocity microphone, there was a very marked distance sensation, whilst in case of the pressure microphone only an increase in the loudness seemed to produce a certain kind of distance effect. A velocity receiver is thus one of the requirements for good distance impression. In contradiction to this seems the fact that with the ears closed with the fingers, there still remains a distance sensation. To obtain similar conditions the microphone was placed into a vessel of water, 1 cm below the surface. The distance effect still remained, showing that the ear makes use of a small component of velocity reception.

The microphone currents can be integrated or differentiated by means of a condenser resistance network. This alters the relative amplitudes of the two terms on the right-hand side of (3) and a change of apparent distance will be expected, provided that the term $(1/r^2)f(t)$ is still sensible at the particular distance from the source. In fact, with the switch in the integrating position, the apparent distance of the source could be decreased from 50 cm to 8 cm, and *vice versa* with the switch in the differentiating position. This phenomenon is so pronounced, that there seems to be no doubt that the relative proportion of the volume flow $f(t)$ and its time differential $f'(t)$ are the factors determining the distance effect. There will be no proper distance sensation, if

$$f(t)=f'(t)$$

as is the case for sinusoidal vibrations (the phase displacement being unimportant).

As continuous tones do not produce a proper distance sensation, only pulses can be made responsible for this effect. As example consider a spark discharge (the oscillograms seem to be sufficiently popular to omit description). Harmonic analysis shows that the pressure time curve $f'(t)$ is lacking of low harmonics in comparison to its integral $f(t)$. In the proximity of the source, where velocity reception is important, the low harmonics will therefore appear. They are not present at larger distances as can be concluded from (3). Observations with a bandpass filter showed that in fact, the high frequencies seem to originate from large distances (60 cm) whilst the low frequencies (150–300 c/sec.) seem to be generated very close to the ear (at about 2 cm).

There seems to be the following physiological explanation for the distance effect. If the stirrup starts to move according to $f(t)$, a wave front will travel along the basilar membrane. The smaller the duration of the impulse, the smaller will be the ballistic deflection of the portion of the basilar membrane which is next to the helicotrema. On the other hand, the part of the basilar membrane in the neighborhood of the stirrup is tuned to the high frequencies and consequently will follow the motion of the stirrup. The shorter the impulse, the nearer to the stirrup will be the centroid of the motion of the basilar membrane. If the stirrup moves according to $f'(t)$, the motion of the basilar membrane will be still closer to the stirrup, the ballistic deflection near the helicotrema becoming zero. The distance effect thus seems to be produced by the change of the position of the "center of gravity" of the motion of the basilar membrane, during a short motion of the stirrup. For a continuous vibration, there is no ballistic deflection and consequently no distance effect.

The human ear allows a certain amount of velocity reception as the impedance measurements of Tröger showed, in the range 800–2000 c/sec. the ear drum impedance reduces to several times the value of the acoustical impedance of the air. It is assumed in general that in the open the distance of every sound source can be localized correctly. This is not true as could be shown with impulse measurements. According to the frequency spectrum of the pulses, the apparent distance varied between 5 and 100 cm.

E. J. SKUDRZYK

Disturbance of Sound Waves Produced by a Rigid Sphere. H. STENZEL, E. N. T. **15**, 71–78 (1938).—The effect of a spherical obstacle on the sound waves has been investigated by Lord Rayleigh. He obtained the following solution for the disturbance due to the presence of the sphere

$$\psi=-\frac{kc^2}{r}e^{ik(at-r+c)}\sum\frac{(2n+1)P_n(\mu)}{F_n(ikr)}f_n(ikr)\times P_n\left(\frac{d}{d\cdot ikr}\right)\frac{d}{d\cdot kc}\frac{\sin kc}{kc}, \tag{1}$$

where r is the distance of the point of observation from the center of the sphere, μ the cosine of the angle between the negative direction of propagation of the incident plane wave and r, c the radius of the sphere, a the velocity of sound and $k=2\pi/\lambda$. The above expression is only of practical use when kc is small. To get over this difficulty, the author introduces Bessel functions, using the following abbreviations:

$$S_n(x)=((\pi/2)x)^{\frac{1}{2}}T_{n+\frac{1}{2}}(x),$$
$$C_n(x)=(-1)^n((\pi/2)x)^{\frac{1}{2}}T_{-n-\frac{1}{2}}(x),$$
$$U_n(x)=xS_{n+1}(x)-nS_n(x),$$
$$V_n(x)=xC_{n+1}(x)-nC_n(x).$$

For simplicity kr is assumed large, so that $f_n(ikr)=1$. Then a short calculation leads to the final expression

$$\psi=ce^{ik(at-R-r)}\cdot\frac{1}{kc}\sum_{n=0}^{\infty}(-1)^n(2n+1)P_n(\mu)\times\frac{(U_n(kc))/(V_n(kc))+i(U_n^2(kc))/(V_n^2(kc))}{1+(U_n^2(kc))/(V_n^2(kc))} \quad (2)$$

Here, the incident plane wave $e^{ikr\mu}$ has been replaced by the spherical wave $e^{ik(r\mu-R)}/4\pi R$ originating at a great distance $R(R\gg r)$ from the center of the sphere. The number of terms of (2) required for practical calculations generally is not much greater than $n=kc$. Unfortunately, the functions $S_n(x)$ and $C_n(x)$ are only tabulated for the numbers $x-2, 3, 4\cdots10$. Points in between are required here, as $U_n(x)$ and $V_n(x)$ are of an oscillating character. The author has calculated these for 2,2 2,4 2,6$\cdots$9,8, 20 and 40. They are tabulated in the publication.

Let a disturbance factor Z be defined by

$$\psi=\frac{c}{4\pi rR}e^{ik(at-R-r)}\{R+iT\}=\frac{ce^{ik(at-R-r)}}{4\pi rR}\cdot Ze^{iX}.$$

(Z thus is the ratio of the secondary to primary amplitude.) With the aid of (2) Z can be calculated. In Fig. 1 is represented the variation of Z with kc, for the angle $\vartheta=0$, showing, that the reflection in the direction of the incident wave initially increases rapidly with kc, remaining practically constant and equal to $\frac{1}{2}$ for values of kc larger than 1,2. Fig. 1 represents the angular variation of Z for four

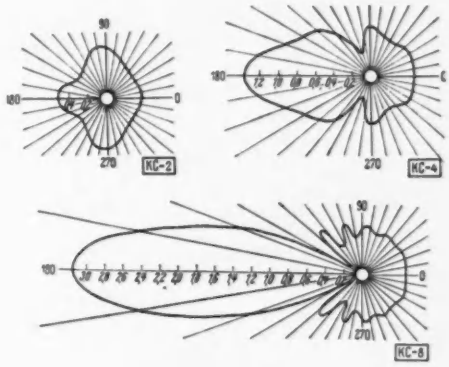

FIG. 1.

different values of kc. In a quite general way, Nicholson has calculated the velocity potential for very large values of kc, transforming the series (1) into a complex integral. In the estimation of the value of the integral, Nicholson however neglected the terms for which $n\geqq kc$. As a calculation for $kc=5$, 10, 20, 40 showed, these terms are important for large values of kc and cannot be neglected.

E. J. SKUDRZYK

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References to Contemporary Papers on Acoustics

MOST of the titles of papers appearing in other languages than English have been translated; the clue as to the language in which the article is written can usually be found by noting the name of the journal in which it appears. Abstracts in English of many foreign papers have appeared or soon will appear in Science Abstracts, Section A. Where references are made to Science Abstracts, the reference is to the volume number and abstract number. The abbreviations of the names of journals are those used in Science Abstracts and can be found in any annual index to those abstracts.

The numbers appearing at the head of each entry are the numbers which identify the subject headings in the Cumulative Index of this journal which is to appear soon.

2. ARCHITECTURAL ACOUSTICS

2.1 **Architectural Acoustics.** P. R. HEYL AND V. L. CHRISLER. Nat. Bur. Stand. Circ. no. C418 (1938) Sci. Abs. 1065B (1938).

2.1 **On the Sound Resonators of the Greek Tragedies and the Christian Church.** P. THIELSCHER. Akustische Zeits. **3**, 85–92 (1938).

2.1 **On a New Procedure for the Measurement of Sound Fields in Interiors by Means of a Continuous Spectrum.** H. G. FREYGANG. Akustische Zeits. **3**, 80–84 (1938).

2.2 **Die Sprechartikulation als Struktur.** P. MENZERATH Forschgn. u. Fortschr. **13**, 364–366 (1937). Phys. Ber. **19**, 385 (1938).

2.3 **Note on the Whispering Gallery of St. Paul's Cathedral, London.** A. E. BATE. Phys. Soc., Proc. **50**, 293–297 (1938).

2.4 **Absorption of Sound by Porous Walls.** A. F. MONNA (In English). Physica **5**, 129–142 (1938).

2.4 **Sound Damping by Air Resonators in Building Construction.** W. ZELLER. Akustische Zeits. **3**, 32–35 (1938).

2.4 **Measurement of the Acoustic Absorption Coefficient.** A. GIGLI. Ingegnere **13**, 4–14 (1938). Sci. Abs. 1419B (1938).

2.4 **The Sound Absorption of Freely Vibrating Plates.** H. LAUFFER. (Book, in German) (Leipzig, 1937).

2.8 **A Reverberation Chamber.** H. A. CHINN. Electronics **11**, 28–29 (1938).

2.9 **The Physical and Technical Basis of Sound Silencing in Building Construction.** S. ARNOLD. (Book, in German) (Hirzel, 1937).

2.9 **The Transmission of Sound in a Building by Indirect Paths.** J. E. R. CONSTABLE. Phys. Soc., Proc. **50**, 368–373 (1938).

2.9 **A Comparative Study of the Sound Insulating Values of Steel Framed and Reinforced Concrete Buildings.** C. W. GLOVER. Structural Eng. **16**, 126–141 (1938).

2.9 **Noise and Vibration Prevention in Buildings.** W. ZELLER. (Book, in German) (V. C. I. Verlag, 1937).

2.9 **The Prevention of the Transmission of Sound Along Water Pipes.** J. E. R. CONSTABLE. Phys. Soc., Proc. **50**, 360–367 (1938).

2.9 **Preventing Sound Transmission.** Sheet Metal Worker **29**, 31–33 (1938).

4. EAR AND HEARING

4.1 **The Sense of Hearing of Fish.** K. VON FRISCH. Nature **141**, 8–11 (1938).

4.1 **The Nature of Fatigue in the Auditory System.** R. C. PARKER. Phys. Soc., Proc. **50**, 108–118 (1938).

4.1 **Absorption Measurements on the Ear Drum with Schuster's Bridge.** E. WAETZMANN. Akustische Zeits. **3**, 1–6 (1938).

4.2 **Electrical Response of the Ear.** E. G. WEVER AND C. W. BRAY. Am. Phil. Soc., Proc. **78**, 407–410 (1937).

4 3 **The Existence of the Faculty of Distance Estimation in Hearing.** G. v. BEKESY. Akustische Zeits. **3**, 21–23 (1938).

4.4 **Hearing Impairment and Sound Intensity.** M. B. GARDNER. Bell Lab. Record **16**, 224–228 (1938).

4.6 **On the Measurement of Absorption and Sensitivity Curves of the Human Ear.** R. KURTZ. Akustische Zeits. **3**, 74–79 (1938).

5. INSTRUMENTS AND APPARATUS. APPLIED ACOUSTICS.

5.1 **Determination of the Resonance Positions of Liquids in Tubes by a Galvanometer Deflection.** C. SALCEANU AND C. ISTRATY. Comptes Rendus **206**, 329–330 (1938).

5.1 **Studies of Free, Sound-Sensitive Flames.** P. E. SCHILLER. Akustische Zeits. **3**, 36–45 (1938).

5.12 **The Measurement of Phase Difference Between Harmonic Oscillations of Different Frequencies.** E. SCHEGOLEV. (In English.) Techn. Phys., U. S. S. R. **4**, 837–840 (1938).

5.13 **Experience with a 5 K.W. Large Loud Speaker Arrangement on the Castle of Nurnberg.** O. VIERLING. Akustische Zeits. **3**, 93–96 (1938).

5.15 **Electronic Voltmeter with Logarithmic Scale.** M. NUOVO. Ricerca Scientifica **2**, 522–530 (1937). Sci. Abs. 1278 (1938).

5.15 **A Versatile Level Meter.** F. SCHUMANN. Electronics. **11**, 13–14 (1938).

5.15 **Rapid Logarithmic Recorder.** A. Hugony. Alta Frequenza. **7**, 84–112 (1938).

5.16 **A Mobile Sound Recording Channel.** L. T. GOLDSMITH AND B. F. RYAN. Soc. Motion Picture Eng., J. **30**, 219–226 (1938).

5.16 **Recent Technical Developments in the Art of Recording Sound for Motion Pictures.** W. HARRIS. Junior Inst. Eng., J. **48**, 257–282 (1938).

5.16 **Direct Disc Recording.** C. J. LEBEL. Electronics **11**, 22–35 (1938).

5.16 **Demonstration of Stereophonic Recording with Motion Pictures.** J. P. MAXFIELD. Soc. Motion Pictures Eng., J. **30**, 131–135 (1938).

5.16 **Some Brief Experiments on the "Almite Record."** A. MIYATA. Inst. Phys. and Chem. Research, Tokyo, Sci. Papers **34**, 205–40 (1938).

5.16 **The Philips-Miller-System of Sound Recording.** R. VERMEULEN. Akustische Zeits. **3**, 65–73 (1938).

5.16 **Gramophone Discs for Electro-Acoustic Investigations.** F. VOGEL AND A. HAAG. Siemens Rev. **8** (Dec. 1937).

5.16 **Electric Control of Acoustic Reverberation.** M. ADAM. R. G. E. **42**, 305–312 (1937). Sci. Abs. 287B (1938).

5.4 **Electromechanical Filter of Variable Frequency.** E. FABINI-GHIRON. Alta Frequenza **7**, 138–144 (1938). Sci. Abs. 1422B (1938).

5.51 **Notions of Acoustic Applied to Radio Audition.** A. GIGLI. I. E. N. G. F. 17 (1937). Sci. Abs. 1417B (1938).

5.8 **On the Effect of the Magnetic Field in the Gap, and of the Mass of the Moving Coil, on the Frequency Characteristic of an Electrodynamic Horn Loudspeaker.** S. A. MAKOV. Izvestiya Elektroprom. Slab. Toka no. 11, 32–35 (1937); Wireless Engineer **15**, 226 (1938).

5.8 **Some Practical Data Relating to Electro-Acoustic Transformers.** M. ADAM. R. G. E. **43**, 47–52 (L938). Sci. Abs. 897B (1938).

5.8 **Radio Progress During 1937.** Electroacoustics. Report by the Technical Committee on Electroacoustics. I. R. E., Proc. **26**, 277–284 (1938).

5.9 **Indetermination of Action of Systems Formed of a Large Number of Associated Elementary Contacts, Such as the Microphone or the Coherer.** A. GUILLET. Comptes Rendus **206**, 333–335 (1938).

6. MUSICAL INSTRUMENTS AND MUSIC

6.1 **Electronic Music Instruments.** Communications **18**, 33–35 (1938).

6.1 **A Textbook of European Musical Instruments, Their Origin, History and Character.** F. W. GALPIN (Book) (Williams and Norgate, 1937).

6.1 **The Measurement of the Frequencies of the Note A in Radio Music.** W. LOTTERMOSER. Akustische Zeits. **3**, 60–61 (1938).

6.5 **Organ Pipes and Edge Tones.** G. B. BROWN. Nature **141**, 11–13 (1938).

6.5 **The Influence of the Material of Metal Organ Pipes on their Tonal Qualities.** W. LOTTERMOSER. Akustische Zeits. **3**, 63–64 (1938).

6.5 **Sound Transitions in the Organ.** F. TRENDELENBURG, E. THIENHAUS AND E. FRANZ. Akustische Zeits. **3**, 7–20 (1938).

6.7 **The Violin.** P. JARNAK. Frank. Inst., J. **225**, 315–341 (1938).

7. NOISE

7.1 **Contribution to the Problem of Maximum Permissible Noise in Telephony.** K. BRAUN. T. F. T. **26**, 284–286 (1937) Sci. Abs. 1004B (1938).

7.1 **International Agreement Determines Standard Noise Measurement Units.** H. FLETCHER. Industrial Standardization **9**, 18–20 (1938).

7.1 **Noise Measurement and the International Conference on Acoustics.** H. FLETCHER. Bell Lab. Record. **16**, 213–219 (1938).

7.1 **Telephone Transmission in Noisy Surroundings.** W. JAVONSKY. E. T. Z. **58**, 1289–1294 (1937). Sci. Abs. 586B (1938).

7.1 **Methods and Results of Subjective and Objective Noise Measurements. Pt. II.** E. LUBCKE. Naturwiss. **26**, 33–41 (1938).

7.1 **Background Noise Corrections.** L. E. PACKARD. Communications **18**, 24 (1938, Mar.).

7.7 **New Technique for Noise Reduction.** E. J. ABBOTT. S. A. E., J. **42**, 170 (1938).

7.7 **The Cause and Elimination of Noise in Small Motors.** W. R. APPLEMAN. Elec. Engineering **56**, 1359–1367 (1937).

9. SPEECH AND SINGING

9.1 **Clinical Studies in Speech Therapy.** A. H. MCALLISTER. (Book). (Univ. of London, 1937).

9.1 **Time of Closure of Glottis from Records of Vowel Sounds.** F. TRENDELENBURG AND W. TRENDELENBURG. Preuss. Akad. Wiss. Berlin, Ber. **20**, 265–277 (1937). Sci. Abs. 1698 (1937).

9.5 **Vowel Structure.** A. GEMELLI. Pont. Acad. Sci., Comm. **1**, 1–39 (1937). Sci. Abs. 284 (1938).

10. SUPERSONICS

10.1 **Ultrasonics and Biology.** A. DOGNON AND E. H. BIANCANI. (Book in French) (Gauthier-Villars, 1937).

10.1 **Detonation of NCl_3 under the Action of Supersonic Waves.** V. BOBOLEV AND J. CHARITON. Acta Physico chimica. **7**, 416 (1937). Phys. Ber. **19**, 281 (1938).

10.1 **On the Dispersion of Ultrasonics in Liquids.** A. K. DUTTA. Phys. Zeits. **39**, 186–187 (1938).

10.1 **Absorption of Ultrasound in Liquids by an Optical Measuring Method.** H. GROBE. Phys. Zeits. **39**, 333–338 (1938).

10.1 **Industrial Applications of Supersonic Vibrations.** H. FREUNDLICH. Indust. Chemist. **14**, 35 (1938).

10.1 **Charged Colloid Particles in an Ultrasonic Field.** J. J. HERMANS. Phil. Mag. **25**, 426–438 (1938).

10.1 **Luminescence in Liquids under Ultrasonic Treatment.** V. L. LEVSIN AND S. N. RZEVKIN. (In English.) Comptes Rendus (Doklady) de l'Acad. des Sciences, U.S.S.R. **16**, 399–404 (1937). Sci. Abs. 167 (1938).

10.1 **New Problems of Ultrasonics.** H. LUDLOFF. Intern. Kongr. f. Kurzwellen, Kongressband. p. 163–166 (1937). Phys. Ber. **19**, 384 (1938).

10.1 **On the Control of Light by Means of Ultrasonics and Their Use as a Fluorometer.** O. MAERCKS. D. Phys. Ges., Verh. **18**, 62–63 (1937). Phys. Ber. **19**, 447 (1938).

10.1 **Diffraction of Light by Supersonic Waves in Solids.** N. S. N. NATH AND H. MUELLER. Nature **141**, 37 (1938).

10.1 **Supersonic Waves in Gases.** D. RIABOUCHINSKY. Comptes Rendus **205**, 1115–1117 (1937).

10.1 **Supersonics in Relation to Molecular Constitution.** E. G. RICHARDSON. Phys. Soc., Reports **4**, 67–75 (1938).

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10.1 **Effect of Supersonic Wave on the Dust Figure in Liquid Dielectrics.** Y. TORIYAMA AND S. SAWA. Phys.-Math. Soc. Japan, Proc. **19**, 984–987 (1937). Phys. Ber. **19**, 684 (1938).

10.1 **On the Determination of the Elastic Constants of Solid Isotopic Bodies with the Help of Ultrasonic Waves.** A. WALTI. (In German.) Helv. Phys. Acta **11**, 113–139 (1938).

10.1 **Dispersion and the Velocity of Sound in Liquids.** S. PARTHASARATHY. Current Science **6**, 55–56 (1937). Phys. Ber. **19**, 600–601 (1938).

10.1 **On the Theory of the Absorption of Sound in Liquids.** L. I. MANDELSTAM AND M. A. LEONTOWITSCH. Shurnal Technitscheskoi Fisiki (A) **7**, 438–439 (1937). Phys. Ber. **19**, 162 (1938).

10.1 **Measurements on the Absorption of Sound in Carbon Dioxide Gas. Determination of the Relaxation Time for the Vibrational Energy as a Function of the Temperature.** A. VAN ITTERBEEK AND P. MARIENS. (In English.) Physica **5**, 153–160 (1938).

10.2 **New Results from Ultrasonic Waves.** E. HIEDEMANN. Chem. Weedblad **34**, 390–397 (1937).

10.2 **The Determination of the Absorption of Ultrasonic Vibrations in Solid and Liquid Media.** N. F. OTPUSHTSHENNIKOV. (In English.) Phys. Zeits. d. Sowjetunion **12**, 736–744 (1937).

10.2 **Effect of Ultrasonic Waves on Solubility of Colloids. Part III.** N. SATA. Kolloid Zeits. **81**, 182–187 (1937). Sci. Abs. 27 (1938). **Part IV.** N. SATA AND Y. NIWASE. Kolloid Zeits. **81**, 294–300 (1937). Sci. Abs. 415 (1938).

10.2 **Dispersion of Supersonic Waves in Liquids. Parts I and II.** B. G. SPAKOVSKIJ. (In English.) Comptes Rendus (Doklady) de l'Acad. des Sciences, U.S.S.R. **18**, 169–175 (1938). Sci. Abs. 1695 (1938).

10.3 **Vibrating Crystals and Their Use in High Frequency and Ultrasonic Technic.** L. BERGMANN. (Book, in German.) (Teubner, 1937.)

10.3 **Velocity of Sound by Capillary Supersonic Interference.** J. P. CANCE. Comptes Rendus **206**, 504–505 (1938).

11. WAVES AND VIBRATIONS

11.1 **Production of Relaxation Oscillations.** S. BYARD. Wireless Engineer **15**, 252–256 (1938).

11.1 **The Status of Sound Investigation.** F. TRENDELENBURG. Ber 250-Jahrfeier der Leopoldina. p. 115–128 (1937). Phys. Ber. **19**, 384 (1938).

11.1 **A Conspectus of Acoustic, Especially the Problems and Possibilities in Italy.** A. GIGLI. Elettrotechica **24**, 6709676 (1937). Sci. Abs. 1427B (1938).

11.1 **Acoustic Resonance Spectra in Liquids.** C. SALCEANU AND C. ISTRATY. Comptes Rendus **206**, 502–504 (1938).

11.1 **Sound.** E. G. RICHARDSON. Phys. Soc., Reports **4**, 53–66 (1938).

11.3 **The Absorption of Sound Energy in the Range of Tone Frequency in the Suspended Parts of the Atmosphere.** W. NUTHMANN. (Diss., Göttingen, 1937.)

11.3 **The Velocity of Sound in Air.** R. C. COLWELL, A. W. FRIEND AND D. A. MCGRAW. Frank. Inst., J. **225**, 579–593 (1938).

11.3 **Sound Velocity and Chemical Constitution.** S. PARTHASARATHY. Current Science **6**, 322–323 (1938). Sci. Abs. 1696 (1938).

11.3 **Measurement of the Speed of Sound in Liquids by Resonance.** C. SALCEANU. Comptes Rendus **205**, 1219–1221 (1937).

11.3 **Propagation of Sound in Gases.** H. L. SAXTON. J. Chem. Phys. **6**, 30–36 (1938).

11.3 **New Measurements of the Velocity of Sound in Air.** T. J. KUKKAMAKI. Ann. d. Physik **31**, 398–406 (1938).

11.3 **Influence of a Magnetic Field on the Absorption of Sound in Oxygen Gas.** A. VAN ITTERBEEK AND L. THYS. (In English.) Physica **5**, 298–304 (1938).

11.4 **Sound Radiation from a Spherical Vibrator.** A. HIRAYAMA. Nippon Elect. Comm. Eng. No. 8, 536–539 (1937). Sci. Abs. 1428B (1938).

11.4 **Wave Field of a Piezo-Quartz Radiator.** S. N. RZEUKIN. Comptes Rendus (Doklady) De l'Acad. des Sciences U. S. S. R. **16**, 267–270 (1937). Sci. Abs. 279 (1938).

11.6 **Resonance of Liquids in Tubes.** C. SALCEANU AND C. ISTRATY. Comptes Rendus **206**, 329–330 (1938).

11.7 **Acoustic Studies of Anisotropic Membranes.** H. L. ZINTL. (Diss.-Erlanger, 1937.)

11.7 **The Passage of Sound Waves Through Plane-Parallel Plates.** H. REISSNER. (In German.) Helv. Phys. Acta **11**, 140–155 (1938).

11.7 **Resonance Effects in a Non-Linear System with Two Degrees of Freedom.** V. MIGULIN. (In English.) Techn. Phys. U. S. S. R. **4**, 850–865 (1938).

11.7 **Untertone.** (Subharmonics etc.) E. WAETZMANN AND R. KURTZ. Ann. d. Physik **31**, 661–680 (1938).

11.7 **On the Theory of Netztone.** K. O. LEHMANN. (Hab.-Schr.) (Karlsruhe, 1937).

11.7 **Spherical and Cylindrical Resonators.** M. FOUCHÉ. Rev. Fac. des Sc. Univ. d'Istanbul (n.s.) **2**, 197–234 (1937). Phys. Ber. **19**, 601 (1938).

11.7 **Relations Existing between Voltage Impulses of Exponential Form and the Response of an Oscillating Circuit.** R. LAMBERT. I. R. E., Proc. **26**, 372–378 (1938).

11.7 **Parametrical Excitation of Combination Oscillations.** V. LAZAREV. (In English.) Techn. Phys. U. S. S. R. **4**, 885–888 (1937).

11.7 **A Method of Harmonic Analysis.—A Study of the Rotational Frequencies of Screw Propellers.** G. F. PARTRIDGE. Phil. Mag. **25**, 505–539 (1938).

11.7 **Matrix-Operational Methods in Mechanical Vibrations.** L. A. PIPES. Frank. Inst., J. **225**, 343–349 (1938).

11.7 **Vibrations of Free Circular Plates. Part I: Normal Modes, Part II: Compounded Normal Modes. Part III: A Study of Chladni's Original Figures.** M. D. WALLER. Phys. Soc., Proc. **50**, 70–86 (1938).

Program of the Nineteenth Meeting of the Acoustical Society of America

MAY 2 AND 3, 1938

NATIONAL BUREAU OF STANDARDS, WASHINGTON, D. C.

MONDAY, MAY 2, 1938, 10:00 A.M.

1. On Sound Localization. HANS WALLACH, *Swarthmore College (Introduced by E. B. Newman).*—The binaural cues for sound localization are effective only in determining the angle between a given direction of sound and the axis of the ears (lateral angle). Since there is localization in the median plane, a second factor should be found which determines the direction of sound in this dimension.

Head movement during sound localization might yield such a factor. With head movement the lateral angle of the sound changes giving a sequence of lateral angles characterizing the direction of sound geometrically in *both* dimensions.

That sound localization actually works on this principle is proved by experiments in which directions of sound are synthetically produced. The sequence of lateral angles which a certain elevated direction of sound would yield with a certain head movement was determined. While the observer performed this head movement a sound was shifted in the horizontal plane in a way to present the same sequence of lateral angles. The observer perceived the sound in the direction from which the presented sequence of lateral angles was derived instead of in the actually given shifting position.

2. Exploration of Pressure Field Around the Human Head During Speech. H. K. DUNN AND D. W. FARNSWORTH, *Bell Telephone Laboratories.*—The pressure spectrum of average speech has been measured at 80 different positions about the head of a speaker, thus showing the directional properties of the mouth and head as a sound radiator. The direction of strongest radiation was found to the front, but varying from somewhat above the horizontal, to 45° below, depending upon the frequency region studied. The results are directly applicable to problems of microphone placement, and of speech intelligibility in different directions. They also permit a more accurate estimate of the spectrum of total speech power than has been possible by older methods. Pressure measurements close to the lips were found to require special precautions, due to extraneous frequencies introduced when the air-stream from the lips strikes the microphone. For the same reason, reproduction from a close pick-up will sound unnatural, unless similar precautions are observed.

3. An Analysis of Perceptible Overtones in the Voice. DON LEWIS AND WILLIAM H. LICHTE, *State University of Iowa.*—Both oscillographic and phonographic recordings were made of a vocal sound in which five successively prominent partials were produced, in a few seconds, by a singer who meanwhile maintained the sound at a specified pitch level. Representative waves from the period of prominence of each partial were subjected to harmonic analysis, the results showing the physical bases for corresponding auditory experiences. Detailed investigation of the periods of transition from a given perceptibly prominent partial to the succeeding one revealed significant changes in energy distribution. For example, during the transition from the 4th to the 5th partial, there was found to be a 5 db decrease in the intensity level of the 4th, a 20 db increase in the level of the 5th, and a 10 db increase in the level of the 6th, with all other partials remaining essentially constant. The changes were gradual but rapid, and were apparently dependent upon resonance factors. The crucial importance of a given transition in attracting attention to a particular partial and, consequently, in shaping perception has been determined by psychological analysis.

4. The Mobility Method of Computing the Vibration of Linear Mechanical and Acoustical Systems. F. A. FIRESTONE, *University of Michigan. (25 min.)*—It is shown that the conventional mechanical-electrical analogy (velocity across→current through; force through→voltage across) has grave defects which have hindered its general adoption as a method of solving vibration problems. The new analogy however (velocity across→voltage across; force through→current through) can be safely applied in such a natural intuitive manner that it becomes unnecessary to distinguish between the mechanical system and the analogous electrical circuit. The problem solving techniques of electricity are taken over bodily into mechanics with the aid of the following devices: The mechanical systems are diagrammed with the aid of a set of conventionalized two-terminal symbols which picture the springs, mechanical resistors, and masses comprising the system. The letter symbols for velocity, force and mass are changed from the usual v, f and m to e, i, and c, respectively. A complex constant called the "mobility" (ease of motion) is defined as $z=e/i=$velocity/force and it is found that the mobility of the mechanical elements is of a familiar form ($z_c=j\omega l$, $z_r=r$, $z_c=-j/\omega c$) while series and parallel combinations of z's are made by the familiar rules. The solution of the mechanical problem is thus reduced to a determination of the mobilities at different points in the system and the application of the equation $e=iz$. The problem remains a problem in mechanics and there is no reference to electricity.

Not only can forced vibration be computed in this manner, but the normal frequencies and normal modes of free vibration of resistanceless systems can be found. The concept of "acoustical mobility," $Z=E/I=$volume velocity/sound pressure, permits the calculation of mechanical-acoustical systems.

5. A New Acceleration Meter (With Demonstration). HARVEY C. HAYES AND W. F. CURTIS, *Naval Research Laboratory.*—Problems involving the lessening or elimination of vibrations in machines and associated mechanical structures call for methods and means of measuring their amplitude and frequency. Several devices have been developed for this purpose which serve well over some limited range of frequencies and amplitude but they offer little or no aid in solving many problems having to do with ships where extremes in both frequency and amplitude are encountered such as shaft and blade vibrations of superaudible frequency and problems involving roll where the period may exceed 10 seconds. To meet these problems a new type of acceleration meter has been developed. It utilizes the change of electrical impedance of a coil of fine wire wound as a solenoid around a wire of magnetostrictive material when tension in the wire is changed. In practice a known mass is anchored to the moving or vibrating member by such a wire and the accelerating forces acting on this mass are measured in terms of change of electrical impedance of the coil which forms one arm of an impedance bridge. The bridge is energized at high frequency and its output can be amplified to give almost any desired degree of sensitivity.

6. Tracing Distortion in Sound Reproduction from Phonograph Records. J. A. PIERCE AND F. V. HUNT, *Cruft Laboratory, Harvard University. (25 min.)*—When the spherical tip of an ideal reproducer stylus slides over a warped groove surface having a sinusoidal profile, the traced curve is not exactly sinusoidal. An analysis of the harmonic content of the traced curve, similar to that given by Di Toro[1] but avoiding his approximations, is directly applicable to reproduction from vertical-cut records. These results may be applied to reproduction from lateral-cut records by taking the original groove surface as inclined approximately 45° from the horizontal, projecting the traced curve on the horizontal and vertical planes, and adding in proper phase the guidance of the stylus tip by *both* sidewalls. It is shown that there is a residual vertical component of stylus motion ("pinch" effect) and complete cancellation of all even harmonics in the tracing distortion. Computation of the remaining odd harmonics indicates that, when the ideal lateral-cut reproducer characteristics include ideal "following" for vertical motion at signal frequencies, a lateral-cut record may be reproduced with 4 to 10 times less r.m.s. distortion than a similarly recorded vertical-cut record. These results are displayed for convenient reference by contours of constant distortion on a universal chart whose dimensionless coordinates characterize any recording condition, and allow the immediate specification of the maximum permissible recorded amplitude, maximum permissible predistortion of the frequency characteristic, and the required clearance angle of the recording stylus.

[1] M. J. Di Toro, J. Soc. Mot. Pict. Eng. **29**, 493 (1937).

MONDAY, MAY 2, 1938, 2:00 P.M.

A specially conducted tour will be made through the Federal Bureau of Investigation. Following the tour a short talk will be given by the assistant director, Mr. H. H. Clegg, on the subject of "Modern Law Enforcement."

MONDAY, MAY 2, 1938, 6:30 P.M.

Society Dinner at Wardman Park Hotel

Cost per plate—$1.75

MONDAY, MAY 2, 1938, 8:00 P.M.

7. Optical Analogy of a Loudspeaker in a Reverberant Room. F. R. WATSON, *University of Illinois.*—When a speaker talks in a large, reverberant room, the numerous reflections from the various surfaces fill the room with sound that may be regarded as an acoustic "fog." The speaker thus has difficulty in making himself "seen" (heard) through this "fog" by distant auditors. By analogy with an auto in a real fog, he turns on his "acoustic headlights" (loudspeakers) to make his speech "visible" (understood) by auditors. Care must be taken to avoid "acoustic glare" (excessive loudness), which is also likely to produce a "reflection" (echo) from distant walls. This optical analogy has been found helpful in explaining the acoustic effects set up by reverberation in rooms.

8. A Method of Observing Sound Decay and Measuring Reverberation Time. W. N. TUTTLE AND HORATIO W. LAMSON, *General Radio Company.*—The shape of the transconductance *versus* grid bias characteristics of certain vacuum tubes is such that, by employing a discharging capacitor to bias these grids in an audio-amplifier, it is possible to have the over-all gain of the amplifier increase

exponentially with time over a wide range to a very close approximation. This known rate of gain increase can be adjusted to match the average rate of decay of reverberant sound so that deviations from a true exponential decay can be observed directly on a meter or cathode-ray oscillograph as decibel differences. The time constant of the discharging capacitor may then be calibrated directly in reverberation time.

9. Measurement of Absorption in Rooms with Sound Absorbing Ceilings. J. R. POWER, *Bell Telephone Laboratories.*—Serious difficulties have been encountered in attempts to measure the absorption coefficients of sound absorbing ceilings in large offices. An analysis of the sound field is made and it is concluded (1) that the reverberation time formula is usually invalid if the absorption is concentrated on one surface of the room, (2) that the energy density formula is not subject to such a restriction and (3) that under such conditions there is no apparent field method of determining the absorption coefficient that will serve as an accurate check on laboratory measurements.

10. Theoretical Investigation of the Absorption of Sound by Vibrating Materials. ROBERT ROGERS, *University of Illinois* (*Introduced by F. R. Watson*).—The present investigation deals with the absorption of sound energy by materials which owe their absorbing power to the dissipation of energy in internal friction during their vibration. A relation is obtained which enables one to calculate the absorbing power of a plate which may be considered to vibrate as a whole. This condition is fulfilled only at low frequencies but this is the region in which the absorbing power of the material is of interest. From this study one is led to the conclusion that the maximum absorption of the plate occurs at the resonant frequency of the system which is made up of the plate backed by an air tight space. In addition the width of the frequency region in which the plate has appreciable absorption is determined. This region may be narrow but the study indicates how such a material may be modified to give appreciable absorption over a considerable range of frequencies. By changing certain constants of the system the maximum absorption point may be varied at will. Although the above results are obtained only for the absorption at normal incidence, they may be extended to all angles of incidence.

11. Effect of Physical Size on the Directional Characteristics of Uni-Directional and Pressure Gradient Microphones. FRANK MASSA, *RCA Mfg. Co. Inc.*—Theoretical and experimental data are presented showing the effect of finite structural size in modifying the directional characteristics of a uni-directional microphone. When the separation between the pressure and pressure gradient actuated sections is increased beyond a small fraction of a wavelength, there is an appreciable reduction in the solid angle in which the attenuation is very high. The effect of the finite enclosure which terminates the pressure actuated element is to shift the axis of maximum attenuation from an axis of symmetry between the microphone elements at low frequencies to one inclined toward the pressure element at the high frequencies. Computed directional characteristics for an idealized pressure gradient microphone indicate a progressive broadening with increasing frequency. For an actual structure, carefully obtained experimental characteristics show a sharpening effect at medium frequencies and eventually a broadening at high frequencies. The intermediate sharpening is explained by the diffraction of sound which causes a pressure increase at normal incidence to the microphone when its dimensions become equal to a sizable fraction of a wave-length.

12. A Miniature Directional Condenser Microphone for Acoustic Measurements. F. J. WILLIG, *University of Illinois* (*Introduced by F. R. Watson*).—Miniature pressure gradient condenser microphones similar to the type described by Braunmuhl and Weber have been developed to measure various acoustic quantities. They are especially useful because of their directional qualities and small size, some being only a centimeter in diameter and a millimeter thick. They are thus small enough to avoid disturbing the sound field, and are advantageous in experiments where the direction of the sound is important. For example, they can be used to map the sound field in an organ pipe. Preliminary experiments have been made to obtain approximate sound absorption coefficients of small samples of materials, by measuring the sound that is incident and the sound that is reflected from the material. Other obvious applications of the microphone are contemplated.

TUESDAY, MAY 3, 1938, 9:30 A.M.

All papers presented at this session deal with standards for acoustical measurements. As this is one of the most important subjects before the Society at the present time it is hoped that there will be a full discussion of each paper.

13. Standard Requirements for Audiometers. WILBERT F. SNYDER, *National Bureau of Standards.*—There is an increasing interest among various groups today in more exact methods of determination of hearing loss. The instrument known as the audiometer has been found to be the most satisfactory for such measurements. But, as is so frequently the case in the development of instruments in a new field, there is not sufficient uniformity in audiometers of different manufacturers to permit interchange of test data. Two years ago a subcommittee of the ASA Sectional Committee on Acoustical Measurements and Terminology was formed to prepare specifications for audiometers. The

specifications in their present form will be reviewed. Attention will be given to several sections of the specifications wherein there has been considerable variance in audiometers of different manufacturers.

14. A Moving Coil Pistonphone for Measurement of Sound Field Pressures. RALPH P. GLOVER[1] AND BENJAMIN BAUMZWEIGER,[2] *Shure Brothers.*—A standard of low frequency sound-pressure was developed following the general theory of previous investigators. The dynamic pistonphone consists of a chamber about 50 cc in volume and a piston 0.3 cm in diameter driven by a moving coil mechanism, capable of executing strokes of approximately 1.0 cm at frequencies between 20 and 100 c.p.s. The stroke is measured by means of a telescope and scale, and the pressure developed is computed assuming isentropic law. A small crystal microphone is used as a pressure indicator within the pistonphone chamber, and becomes a calibrated microphone for the measurement of sound field pressures. The device appears to be more simple and easier to use than pistonphones previously described in the literature.

[1] Chief Engineer, Shure Brothers, Chicago.
[2] Development Engineer, Shure Brothers, Chicago.

15. Field Calibration of Microphones. G. S. COOK, *National Bureau of Standards (Introduced by V. L. Chrisler).*—The first requirement for the making of field calibration of microphones is a space in which free field conditions are realized as nearly as possible. A room has been constructed 16′×16′×12′. The treatment of the room is the same as that used in a similar room constructed at the Bell Telephone Laboratories with the exception that there is a four-inch layer of rock wool outside the layers of cloth. Exploration of the room shows a marked pattern as was found in the room at the Bell Telephone Laboratories. However, fair conditions are to be had when both the sound source and receiver are near the center of the room. The results of microphone calibration show fair agreement with those of other laboratories. When observations are made at short frequency intervals there are variations in the apparent sensitivity of a microphone sometimes greater than 1 db. These are evidently due to interference. Such variations are to be expected even with walls having a reflecting power of about one percent. It seems desirable to make a group of observations over a short frequency range rather than several observations at the same frequency.

16. Absolute Sound Measurements in Liquids. ELIAS KLEIN, *Naval Research Laboratory.*—The fundamental problem of absolute sound measurements in liquids is discussed. Regarding acoustic determinations, reference is made to certain inherent differences between air and water. An outline is presented of three methods now available. A modified radiation pressure apparatus is described which permits a dual check of the basic measurements involved, and at the same time makes possible microphone calibration for secondary standards.

17. A Calibrator for Sound Level Meters. J. M. BARSTOW, *Bell Telephone Laboratories.*—Experience has shown that a secondary acoustic standard suitable for calibrating sound level meters would be desirable and a certain amount of experimental work has been done toward the development of such a device. A model which generates a sound having a level of 84 db (40 db loudness weighting) and a frequency composition closely simulating that of a general type of room noise has been produced. The apparatus is to be demonstrated and it is suggested that its output be measured with the various sound level meters present as a means of checking the reference points employed.

TUESDAY, MAY 3, 1938, 2:00 P.M.

18. A Study of the Tuning of Pianos. O. L. RAILSBACK, *Eastern Illinois State Teachers College.*—Measurements of the pitch of each note on the piano were made immediately after tuning by means of the chromatic stroboscope.[1] Sixteen grand pianos and twelve upright pianos were included in this study. Different makes of pianos were used and the tuning was done by different tuners. Averages were computed for each of four tuners, using three grand pianos by each tuner, to discover whether or not different tuners follow different patterns of tuning. The conclusion is that they follow, on the average, about the same pattern with individual differences on the extreme ends. With this established the average was then taken of sixteen grand pianos to determine the slope of average deviation from the equally tempered scale. A line drawn to true fifths much more nearly coincides with the line of averages than does the line of true octaves and is very close for the middle five octaves. At the ends the departure is considerably greater. For the twelve upright pianos the departure is still greater. A variety of causes seem to contribute to the "stretching" of octaves. Inharmonic overtones in the bass and over compensating to avoid "flatting" are probably most significant.

[1] J. Acous. Soc. Am. **9**, 37 (1937).

19. An Adjustable Tuning Fork. O. HUGO SCHUCK, *C. G. Conn, Ltd.*—A portable tertiary frequency standard, adjustable over a small range, is described. The frequency determining element is a valve-maintained tuning fork, adjustment of whose frequency over a relatively small range is accomplished by changing the position of movable weights on the prongs. Calculations are simplified by the design of the fork, which permits the assumption of lumped constants. The weights may be moved while the fork is running without interfering with its operation. Indication of their position is by means of a pointer reading on a scale graduated directly in frequency units. A precise method of calibration is given. Mechanical arrangements are described for obtaining a uniformly spaced scale, and the possibility of using

such a scale in correcting for the temperature coefficient of the fork material is discussed. The relative accuracy of such a fork is greater, the smaller the range of frequency adjustment. In the present development, adjustment may be made over a range of plus and minus three percent of the mean frequency, that is, a musical interval of one semi-tone or 100 cents total change. Incorporation of this fork in the chromatic stroboscope has resulted in considerable simplification as well as an increase in the reliability of the instrument.

20. The Departure of the Overtones of a Vibrating String from a True Harmonic Series. ROBERT S. SHANKLAND, *Case School of Applied Science.*—The changes in frequency caused by variations in tension due to the amplitude of vibration of a wire are treated by the method of perturbations. The constraints introduced by the bridges at the end nodes as studied by Raman,[1] and on the bending moments due to the stiffness of the wire are included in the treatment. The overtones do not, in general, form a true harmonic series. In general, the combined influence of stiffness and amplitude sharpens the overtones with respect to the fundamental. A new type of monochord has been designed with which a cathode ray oscillograph is used to determine the effect of amplitude on frequency of strings of various lengths, diameters and materials. The absolute frequency is determined with a thyratron scale-of-sixteen circuit in connection with a Riefler standard clock. The electrically maintained vibrations are essentially free vibrations corresponding to the vibrations of piano wires. The degree of departure to be expected from an exact harmonic series for these instruments is shown to be in agreement with the studies by Miller[2] for piano wires.

[1] C. V. Raman, Phys. Rev. **32**, 307, 309 (1911).
[2] D. C. Miller, Unpublished studies of piano strings.

21. An Optical Harmonic Analyzer. H. C. MONTGOMERY, *Bell Telephone Laboratories.*—An instrument has recently been completed which makes a Fourier analysis of a function by optical means. It is especially adapted to the analysis of speech and music, since it operates directly from a conventional type of sound track on film. The first 30 harmonics can be measured, the time required being a little over one minute. A similar analysis by methods previously available would require several hours. The instrument will handle fundamental frequencies from 65 to 310 cycles, and the amplitude range is about 35 db. A description of the construction and operation of the instrument will be given, together with a discussion of its capabilities and examples of its work.

22. Recent Advances in the Use of Acoustic Instruments for Routine Production Testing. BLAIR FOULDS, *Electrical Research Products, Inc.*—Early attempts to use sound level meters for routine inspection purposes were not generally satisfactory. Subsequent experience in such applications has uncovered the basic reasons for early failures. These reasons are discussed and solutions reached in certain projects described. Practical considerations in the use of such equipment under factory conditions are discussed.

23. Acoustic Effects of Humidity in Gases. W. H. PIELEMEIER, *The Pennsylvania State College.*—Small anomalous absorption and velocity variations in air produced by water vapor were reported last year.[1] Since then, Kendig has investigated oxygen in a similar way in our laboratory. The anomalous velocity effect was likewise small but absorption peaks were found at approximately the humidity predicted by extrapolating to 98 and 284 kc/sec. on Knudsen's[2] curve. Extensive work has now been done with CO_2. At 528 kc/sec. ($T=300°A$) the partial vapor pressure (p) was changed by 55 steps. The absorption per cm (m) rose rapidly in the water vapor pressure region 1 mm to 4.2 mm. The velocity went through an inverted S-curve for which the steepest portion came at $p=4.2$ mm as it should. The total decrease was about 9 m/sec. A second frequency (1080 kc/sec.) yielded similar curves. The peak absorption is at $p=7.5$ mm. The two peak values are proportional within 1 percent to the respective frequencies, in accordance with the theory. The data show a great effect of water vapor on the average life time of a quantum of vibrational energy in the CO_2 molecules and suggest slight traces of water vapor as the cause of the variously reported critical frequency (45 to 200 kc/sec.) in "dry" CO_2.

[1] W. H. Pielemeier, Phys. Rev. **52**, 244 (1937).
[2] V. O. Knudsen and L. Obert, J. Acous. Soc. Am. **7**, 249 (1936).

24. Theory of Acoustic Effects of Humidity in CO_2. HAROLD L. SAXTON, *The Pennsylvania State College (Introduced by W. H. Pielemeier).*—Letting h represent the ratio of water vapor molecules to total number of molecules in a mixture of CO_2 with water vapor, the collision constant, S,[1] is given by

$$S=a_0+a_1h+a_2h^2.$$

At an absorption peak, S can be calculated from a theoretical formula and h measured. By using principally the data of Pielemeier at 528 and 1080 kc/sec. the constants have been determined. When S is thus expressed in terms of h, the velocity versus h curve, with frequency and temperature held constant, can be predicted with considerable precision. The complete data of Pielemeier at either frequency afford an excellent check on the theory.

[1] H. L. Saxton, J. Chem. Phys. **6** (30), 1938.

25. The Crevasse Phenomenon in Piezoelectric Quartz and its Application in Physical Measurements. J. C. HUBBARD, *The Johns Hopkins University.*—Piezoelectric quartz resonators, because of their mechanical durability and their small decrement, have found many well-known uses, particularly in frequency stabilization and in the construction of electric filters for high frequency channels, but their possibilities in connection with general laboratory use have not been realized. A piezoelectric ultramicrometer has been described[1] permitting linear displacements to be measured to 10^{-9} cm under ordinary laboratory conditions. Work has been continued on the application of this method to the precise measurement of the thermal coefficients of expansion of fine wires, the diameter of fibers, the effect of small temperature and pressure changes on the dielectric

constants of gases, the estimation of moisture content in thin solid films, and the dielectric constants of thin solid and liquid films.

[1] J. C. Hubbard, Phys. Rev. **49**, 865 (1936); Science **83**, 472 (1936).

26. Acoustic Absorption and Reflection Coefficients in Gases by Supersonic Interferometry. RAY S. ALLEMAN, *The Johns Hopkins University* (*Introduced by J. C. Hubbard*).—The necessary experimental requirements of acoustic interferometry in gases have been examined by optical and electrical methods with the object of evaluating the influence of the boundary of the acoustic field, and the effect of departure from parallelism of the reflector with the source upon the experimentally determined attenuation and reflection coefficients. The results show that the requirements of parallelism of the reflector are as rigorous as in optical interferometry and are best met by adjustment for maximum height of peaks rather than by optical or mechanical methods. Recently determined values of reflection coefficients at 1237 kc are found to be in much better agreement among themselves and with the theoretical predictions of the theory (in press) of K. F. Herzfeld for dissipative reflection coefficients than most of those heretofore obtained with the resonator interferometer.[1] Schlieren photographs of modes of vibration of X-cut piezoelectric quartz plates and of the acoustic radiation from them will be shown.

[1] J. C. Hubbard, Phys. Rev. **41**, 523 (1932); R. W. Curtis, Phys. Rev. **46**, 811 (1934).

27. The Absorption of Vibrating Plates. H. A. LEEDY, *University of Illinois*.—The problem of obtaining materials having higher absorptivities at the low frequencies has lead to a theoretical and experimental investigation of the absorptivities of vibrating plates. The absorptivities of such materials are theoretically determined by what may be called a pure "impedance method." This theory shows that maximum absorption occurs at the natural frequencies of the system composed of the vibrating plate and air space. The absorptivities, as determined by the reverberation room method, of $\frac{1}{8}$ in. Celotex board backed by 3 in. and 6 in. air spaces are found to be in satisfactory agreement with the theory. The addition of a layer of $\frac{1}{4}$ in. of cotton on the Celotex board is found to increase materially the width of the frequency range in which the material has relatively large absorption, thus indicating the addition of a considerable amount of damping resistance.

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